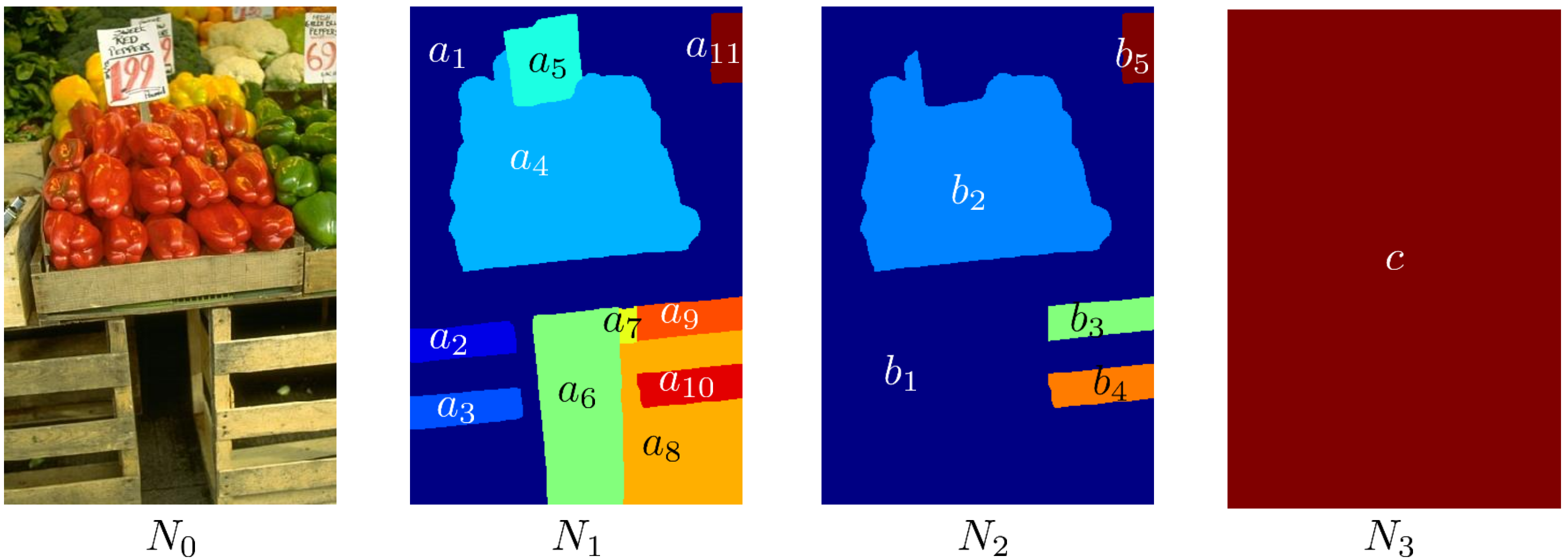
Multi-labeling Optimization on Hierarchies of Partitions

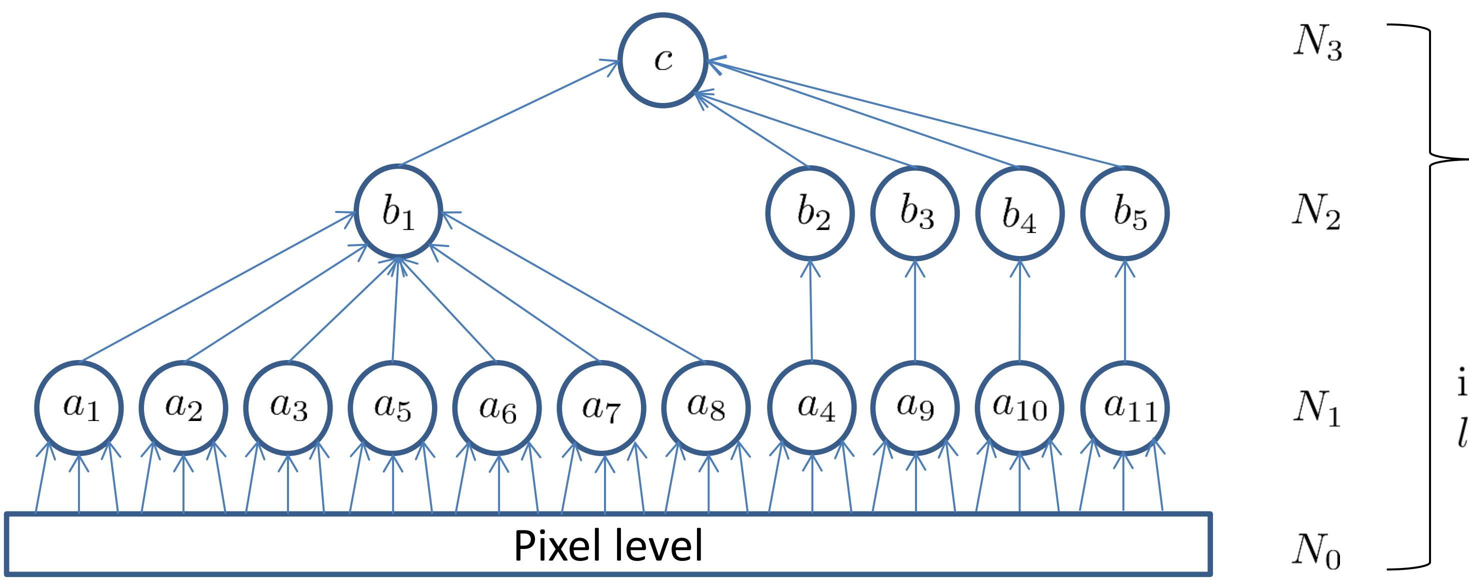
B. Ravi Kiran
Frank R. Schmidt
Jean Serra

ESIEE 05 June 2014

Results

Hierarchy of Labels



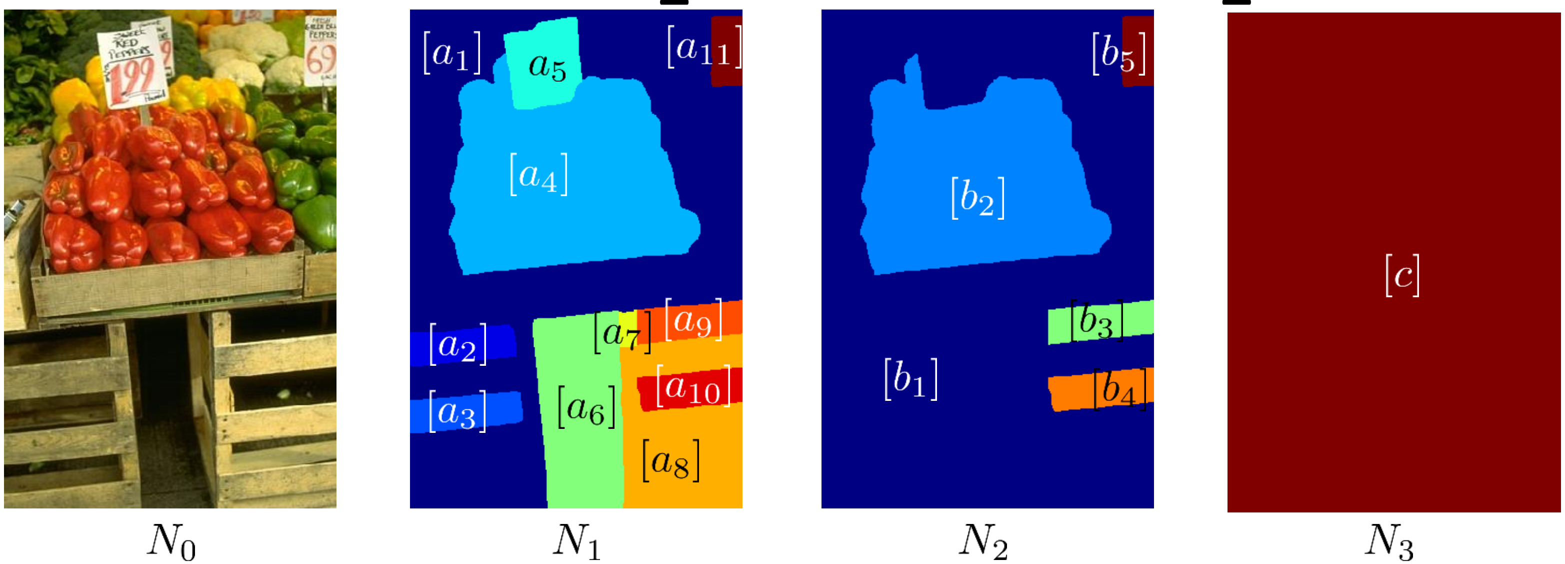


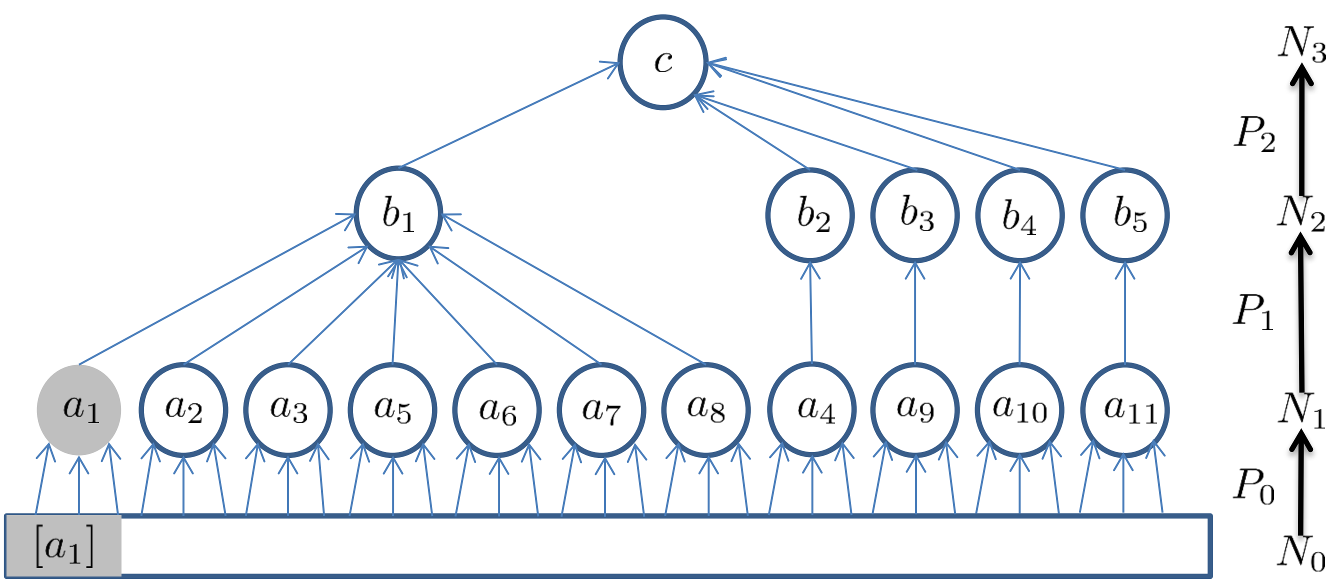
$$N := \bigsqcup_{i=0}^{r-1} N_i$$

$$\overline{N} := N \sqcup N_r$$

$\mathrm{idx}(l) = i$, where
$l \in N_i, i \in \{1, 2, ..., r\}$

Class [a$_1$] of Label a$_1$



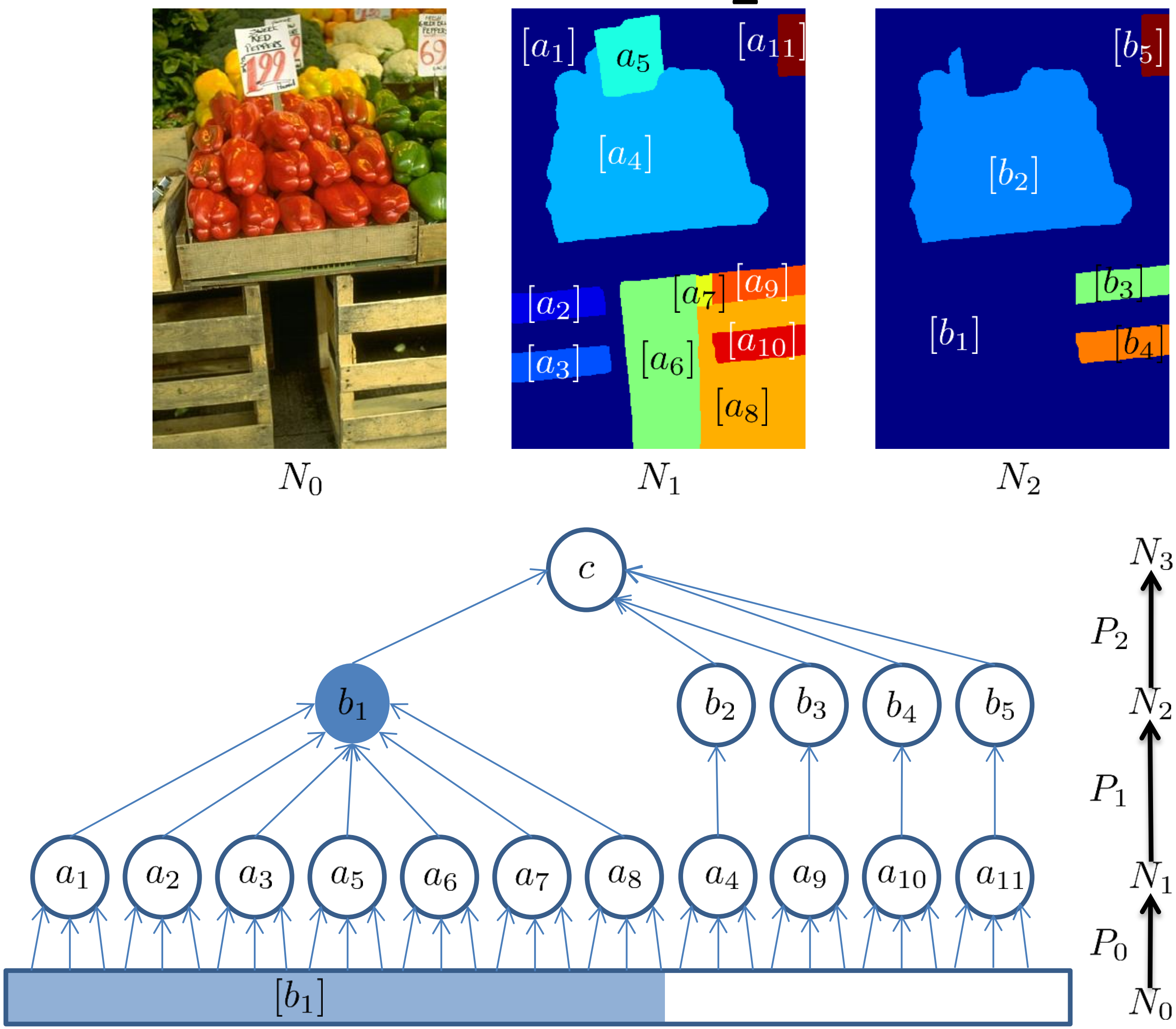


$$P : N \to \bar{N}$$

$$P|N_i = P_i$$

$$[l] = \begin{cases} \{l\}, & \text{if } \ l \in N_0 \\ P^{-i}(l), & \text{if } \ l \in N_i \end{cases}$$

Class [b$_1$] of Label b$_1$

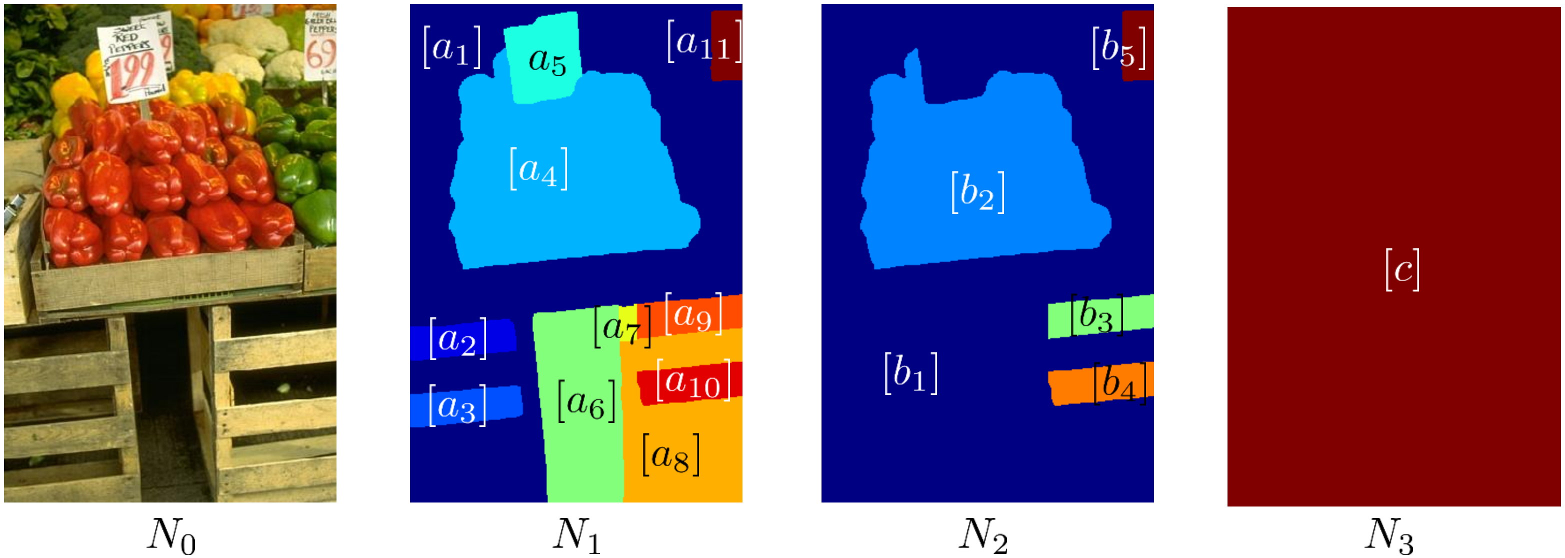


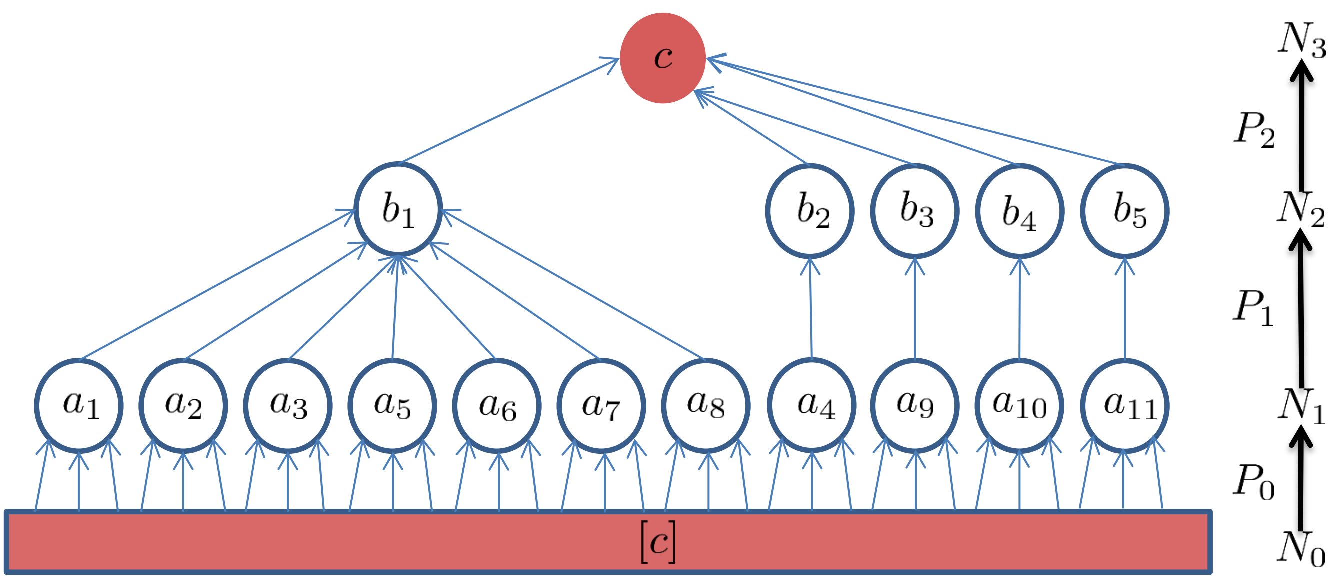
$$P : N \to \bar{N}$$

$$P|N_i = P_i$$

$$[l] = \begin{cases} \{l\}, & \text{if } l \in N_0 \\ P^{-i}(l), & \text{if } l \in N_i \end{cases}$$

Class [c] of Label c



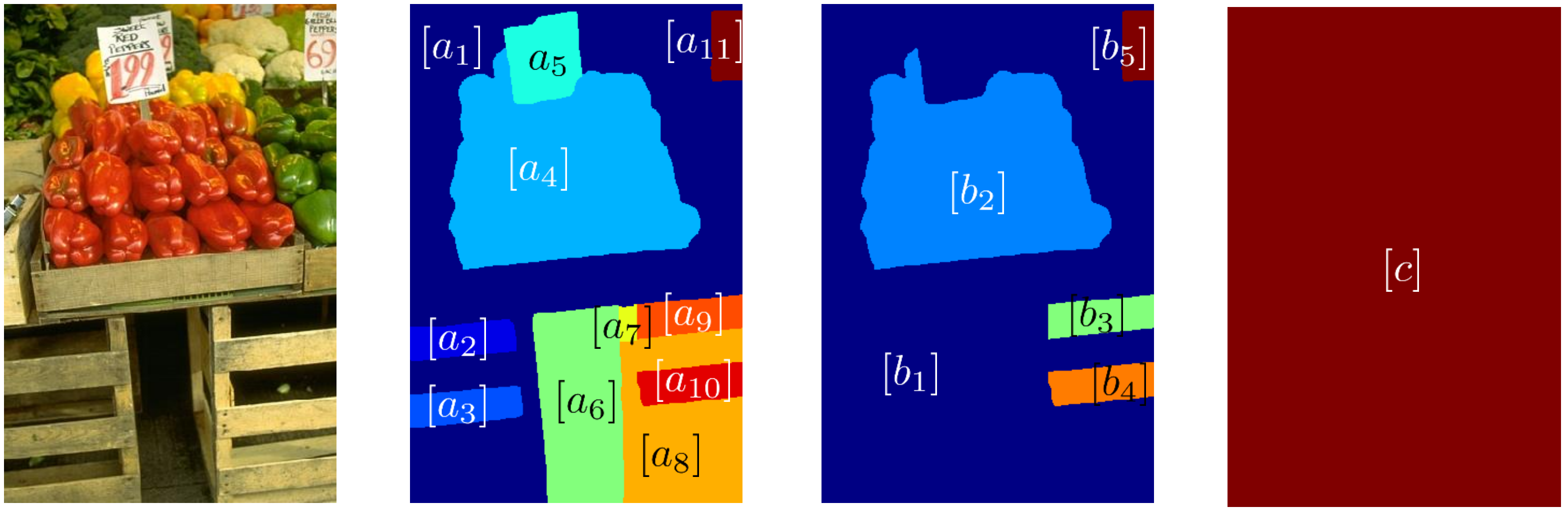


$$P : N \to \bar{N}$$

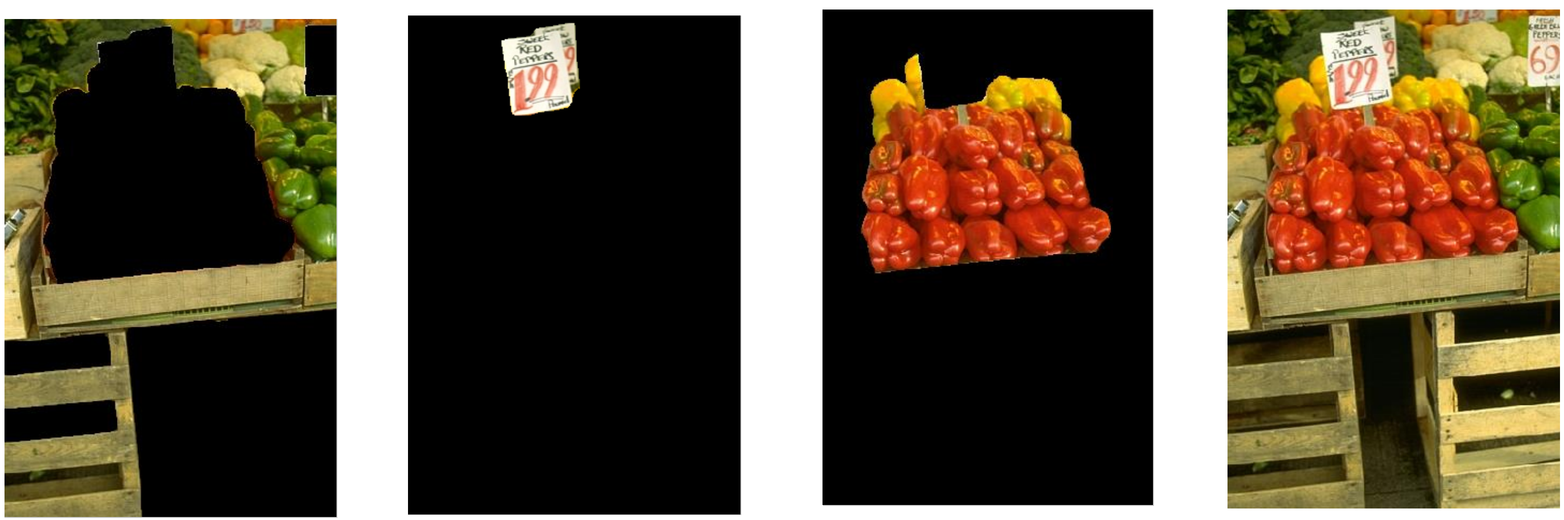
$$P|N_i = P_i$$

$$[l] = \begin{cases} \{l\}, & \text{if } l \in N_0 \\ P^{-i}(l), & \text{if } l \in N_i \end{cases}$$

Selected Classes



N_0 N_1 N_2 N_3



$[a_1]$ $[a_5]$ $[b_2]$ $[c]$

Global Energy

$$E(P) = \sum_{i=1}^{r} \left[\sum_{l \in N_i} \int_{[l]} f_l(x) \mathrm{d}x + \lambda_i \sum_{l < m \in N_i} d(l,m) \cdot \mathrm{Length}(\partial[l] \cap \partial[m]) \right]$$

- f_l is the negative log-likelihood of a probabilistic model
- $d(l,m) := \mathrm{idx}(l \vee m) - \mathrm{idx}(l) = d(m,l)$
- Length is weighted by $d(l,m)$ the distance to their maximum.

Global Energy

$$E(P) = \sum_{i=1}^{r} \Bigg[\underbrace{\sum_{l \in N_i} \int_{[l]} f_l(x) \mathrm{d}x}_{\text{Data term}} + \lambda_i \underbrace{\sum_{l<m \in N_i} d(l,m) \cdot \mathrm{Length}(\partial[l] \cap \partial[m])}_{\text{Length term}} \Bigg]$$

Global Energy

$$E(P) = \sum_{i=1}^{r} \Bigg[\underbrace{\sum_{l \in N_i} \int_{[l]} f_l(x) \mathrm{d}x}_{\text{Data term}} + \lambda_i \underbrace{\sum_{l<m \in N_i} d(l,m) \cdot \mathrm{Length}(\partial[l] \cap \partial[m])}_{\text{Length term}} \Bigg]$$

Geodesic Length: $\mathrm{Len}_{l,m}(s) = d(l,m) \cdot g_{l,m}(s)$

$$E(P) = \sum_{i=0}^{r-1} \sum_{u \in N_i} \int_{[u]} f_{P_i(u)}(x) \mathrm{d}x + \sum_{i=1}^{r} \lambda_i \cdot \sum_{l<m \in N_i} \int_{\partial[l] \cap \partial[m]} \mathrm{Len}_{l,m}(s) \mathrm{d}s$$

Global Energy

$$E(P) = \sum_{i=1}^{r} \Bigg[\underbrace{\sum_{l \in N_i} \int_{[l]} f_l(x) \mathrm{d}x}_{\text{Data term}} + \lambda_i \underbrace{\sum_{l < m \in N_i} d(l,m) \cdot \mathrm{Length}(\partial[l] \cap \partial[m])}_{\text{Length term}} \Bigg]$$

Geodesic Length: $\mathrm{Len}_{l,m}(s) = d(l,m) \cdot g_{l,m}(s)$

$$E(P) = \sum_{i=0}^{r-1} \sum_{u \in N_i} \int_{[u]} f_{P_i(u)}(x) \mathrm{d}x + \sum_{i=1}^{r} \lambda_i \cdot \sum_{l < m \in N_i} \int_{\partial[l] \cap \partial[m]} \mathrm{Len}_{l,m}(s) \mathrm{d}s$$

$$D_u(l) = \int_{[u]} f_l(x) \mathrm{d}x$$

Neighbourhoods $\mathcal{N}_i \subset N_i \times N_i$

$$E(P) = \sum_{i=0}^{r-1} \Bigg[\sum_{u \in N_i} D_u(P_i(u)) \quad + \quad \lambda_i \sum_{u,v \in \mathcal{N}_i} V_{u,v}(P_i(u), P_i(v)) \Bigg]$$

Global Energy

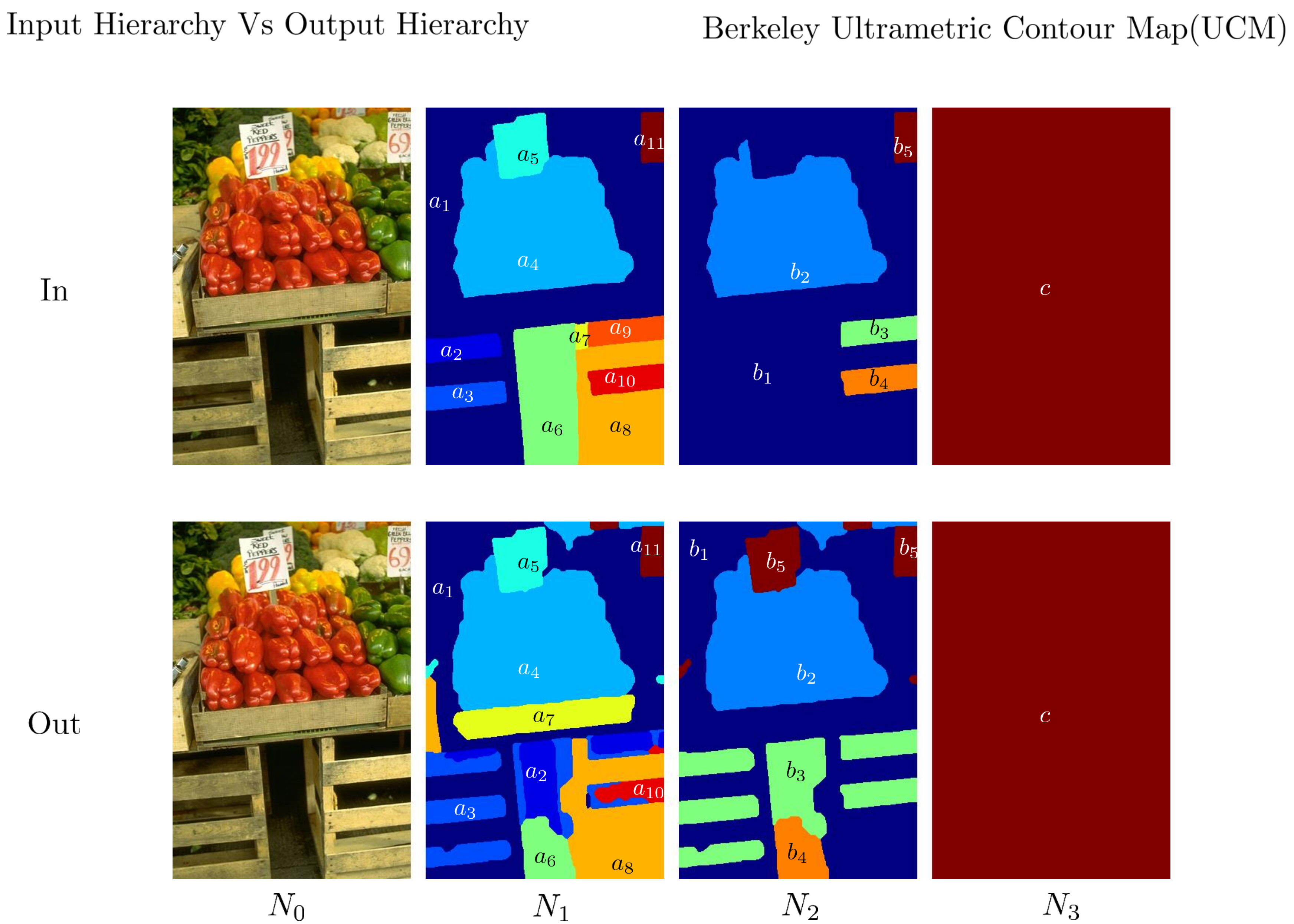
$$E(P) = \sum_{i=0}^{r-1} \left[\sum_{u \in N_i} D_u(P_i(u)) \quad + \quad \lambda_i \sum_{u,v \in \mathcal{N}_i} V_{u,v}(P_i(u), P_i(v)) \right]$$

$$= \sum_{i=0}^{r-1} \quad E_i(P_i)$$

$$E_i(P_i) = \left[\sum_{u \in N_i} D_u(P_i(u)) \quad + \quad \lambda_i \sum_{u,v \in \mathcal{N}_i} V_{u,v}(P_i(u), P_i(v)) \right]$$

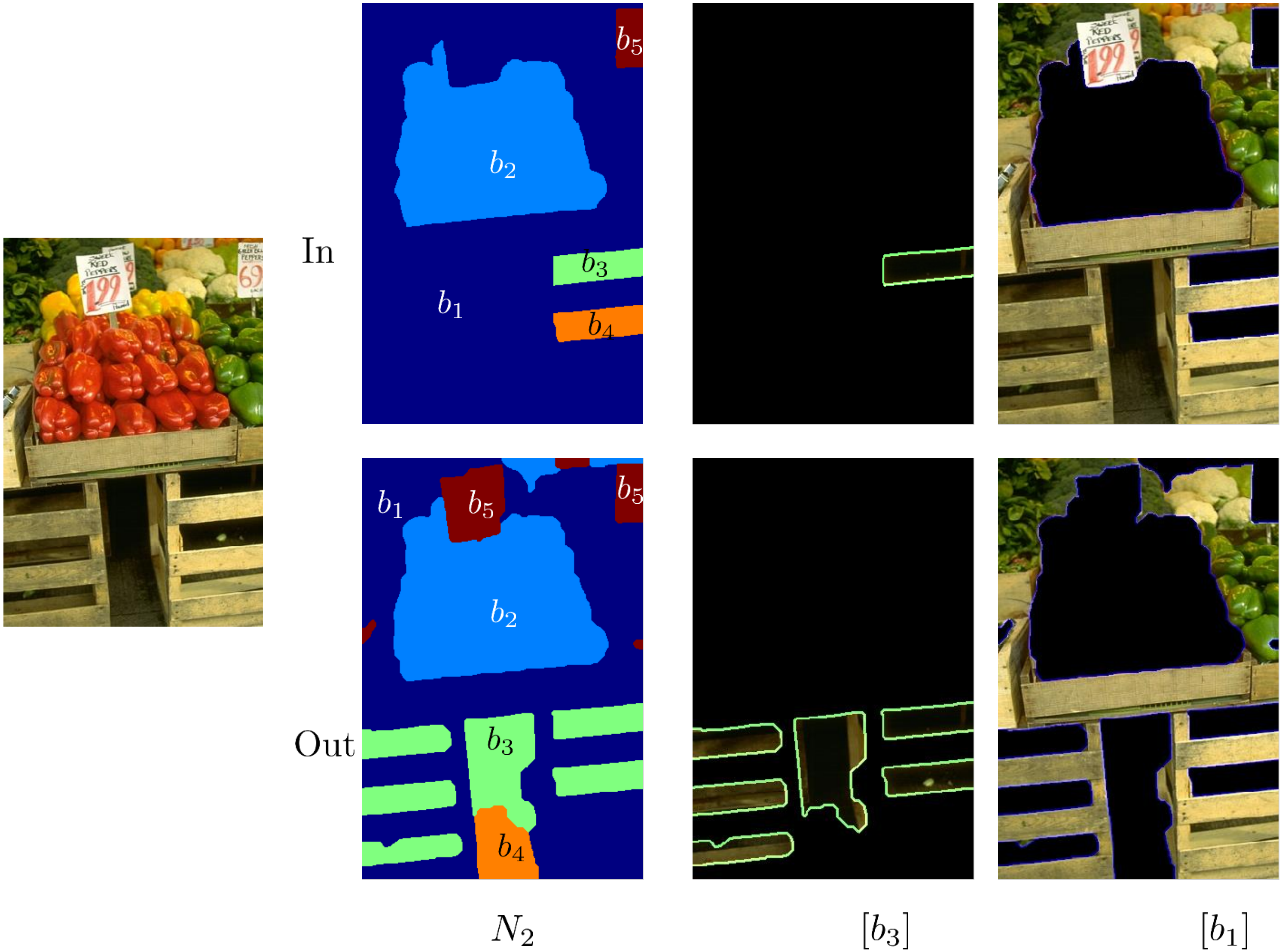
E_i can be optimized via α-expansion.

This results in a new hierarchy $\hat{P}$ with $E(\hat{P}) \leq E(P)$.



Input Hierarchy Vs Output Hierarchy
Berkeley Ultrametric Contour Map(UCM)
In
Out
a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11}
b_1 b_2 b_3 b_4 b_5
c
N_0
N_1
N_2
N_3

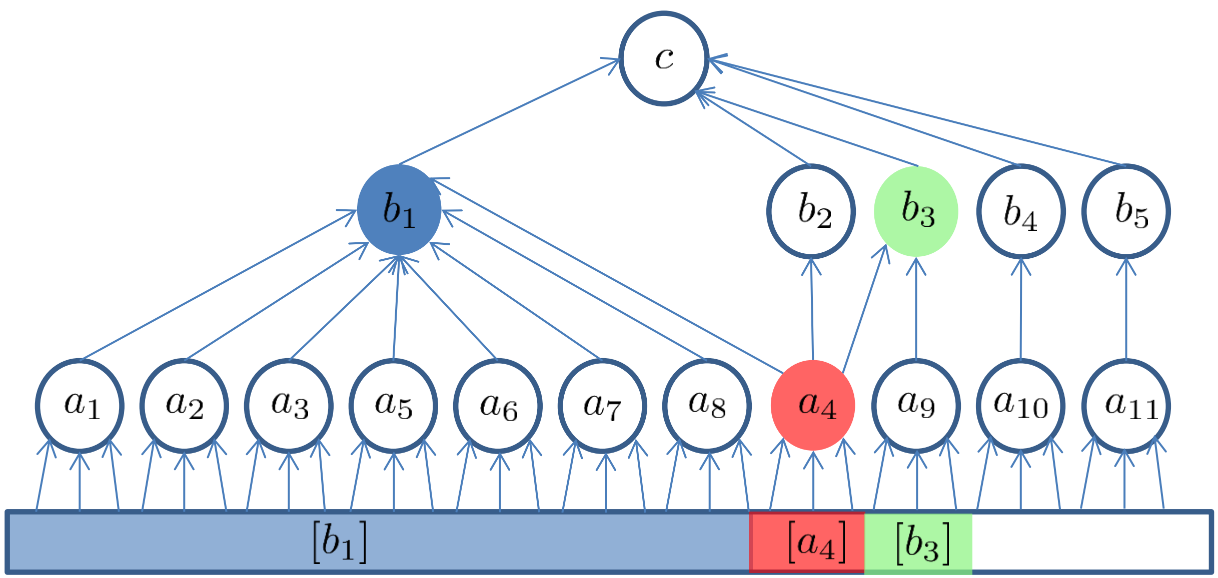
Before and After Optimization: Selected Classes



Parent Label Costs

$$D_u(l) = \int_{[u]} f_l(x)\mathrm{d}x$$

$$D_{a_4}(b_1) = \int_{[a_4]} -\log f_{b_1}(x)\mathrm{d}x \qquad D_{a_4}(b_3) = \int_{[a_4]} -\log f_{b_3}(x)\mathrm{d}x,$$



Hierarchical Structure Costs

$$d(l, m) = \mathrm{idx}(l \vee m) - \mathrm{idx}(l)$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
a_1	0	1	1	2	1	1	1	1	2	2	2
a_2	1	0	1	2	1	1	1	1	2	2	2
a_3	1	1	0	2	1	1	1	1	2	2	2
a_4	2	2	2	0	2	2	2	2	2	2	2
a_5	1	1	1	2	0	1	1	1	2	2	2
a_6	1	1	1	2	1	0	1	1	2	2	2
a_7	1	1	1	2	1	1	0	1	2	2	2
a_8	1	1	1	2	1	1	1	0	2	2	2
a_9	2	2	2	2	2	2	2	2	0	2	2
a_{10}	2	2	2	2	2	2	2	2	2	0	2
a_{11}	2	2	2	2	2	2	2	2	2	2	0

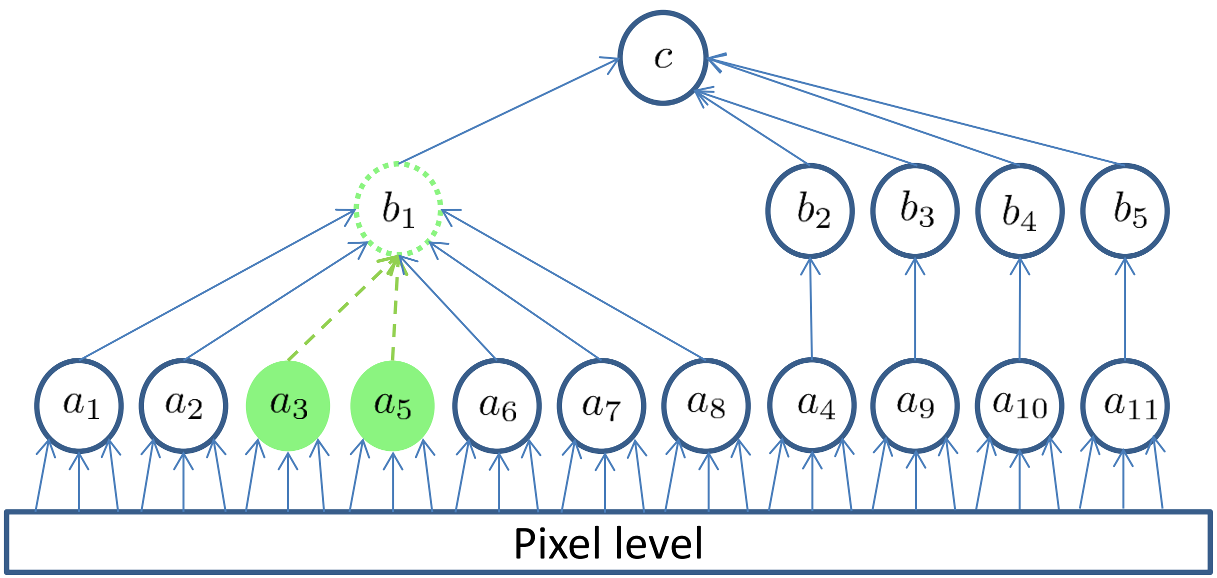
d_{N_1}

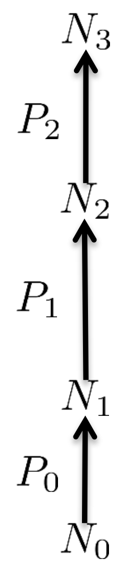
	b_1	b_2	b_3	b_4	b_5
b_1	0	1	1	1	1
b_2	1	0	1	1	1
b_3	1	1	0	1	1
b_4	1	1	1	0	1
b_5	1	1	1	1	0

d_{N_2}

	c
c	0

d_{N_3}





$$P : N \to N$$

$$P|N_i = P_i$$

$$[l] = \begin{cases} \{l\}, & \text{if } l \in N_0 \\ P^{-i}(l), & \text{if } l \in N_i \end{cases}$$

Hierarchical Structure Costs

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
a_1	0	1	1	2	1	1	1	1	2	2	2
a_2	1	0	1	2	1	1	1	1	2	2	2
a_3	1	1	0	2	1	1	1	1	2	2	2
a_4	2	2	2	0	2	2	2	2	2	2	2
a_5	1	1	1	2	0	1	1	1	2	2	2
a_6	1	1	1	2	1	0	1	1	2	2	2
a_7	1	1	1	2	1	1	0	1	2	2	2
a_8	1	1	1	2	1	1	1	0	2	2	2
a_9	2	2	2	2	2	2	2	2	0	2	2
a_{10}	2	2	2	2	2	2	2	2	2	0	2
a_{11}	2	2	2	2	2	2	2	2	2	2	0

d_{N_1}

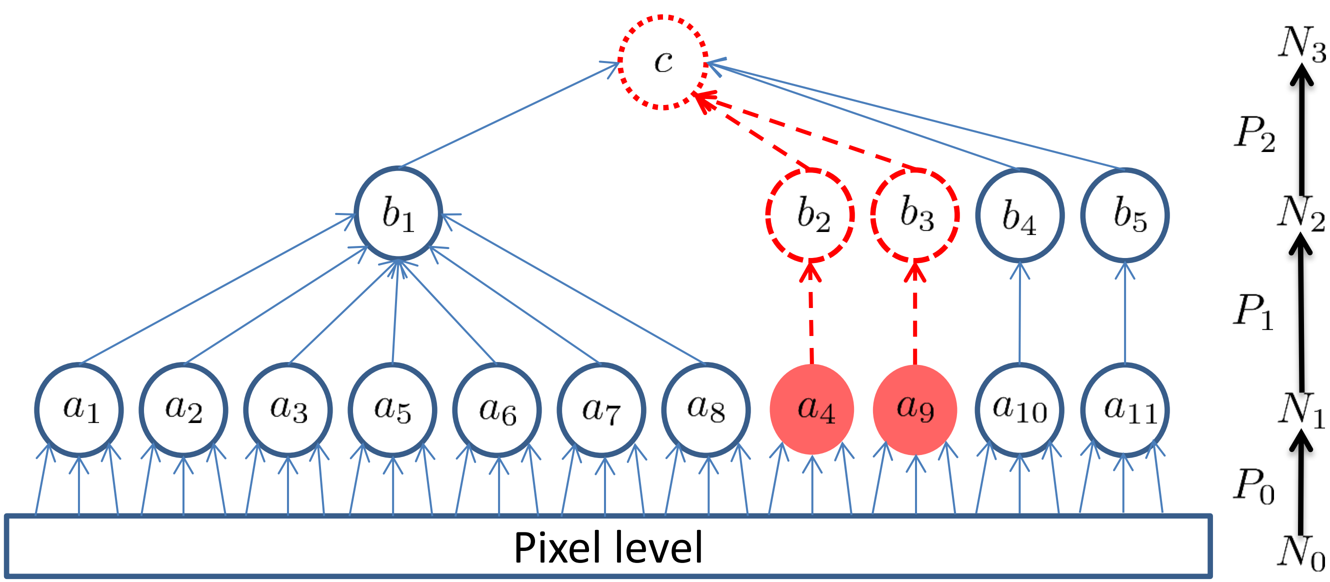
	b_1	b_2	b_3	b_4	b_5
b_1	0	1	1	1	1
b_2	1	0	1	1	1
b_3	1	1	0	1	1
b_4	1	1	1	0	1
b_5	1	1	1	1	0

d_{N_2}

	c
c	0

d_{N_3}

$$d(l,m) = \mathrm{idx}(l \vee m) - \mathrm{idx}(l)$$

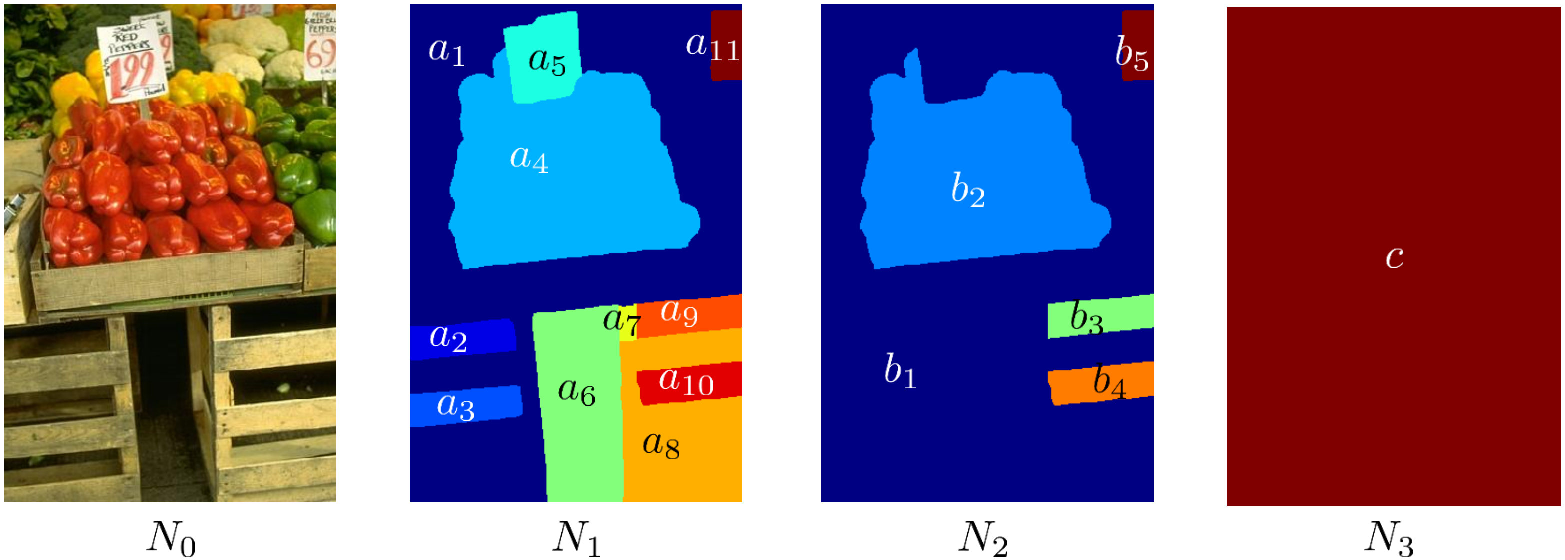


$$P : N \to N$$

$$P|N_i = P_i$$

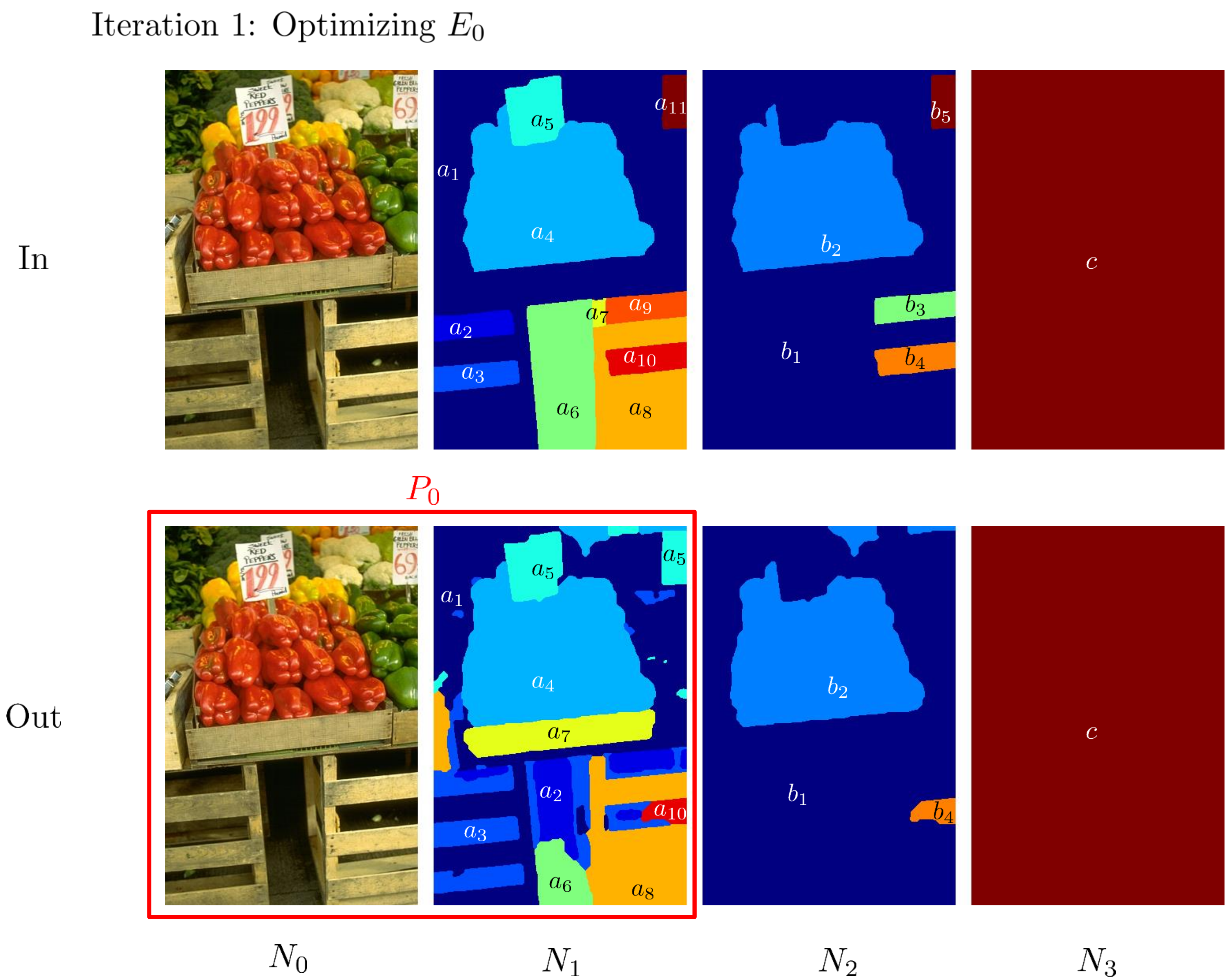
$$[l] = \begin{cases} \{l\}, & \text{if } l \in N_0 \\ P^{-i}(l), & \text{if } l \in N_i \end{cases}$$

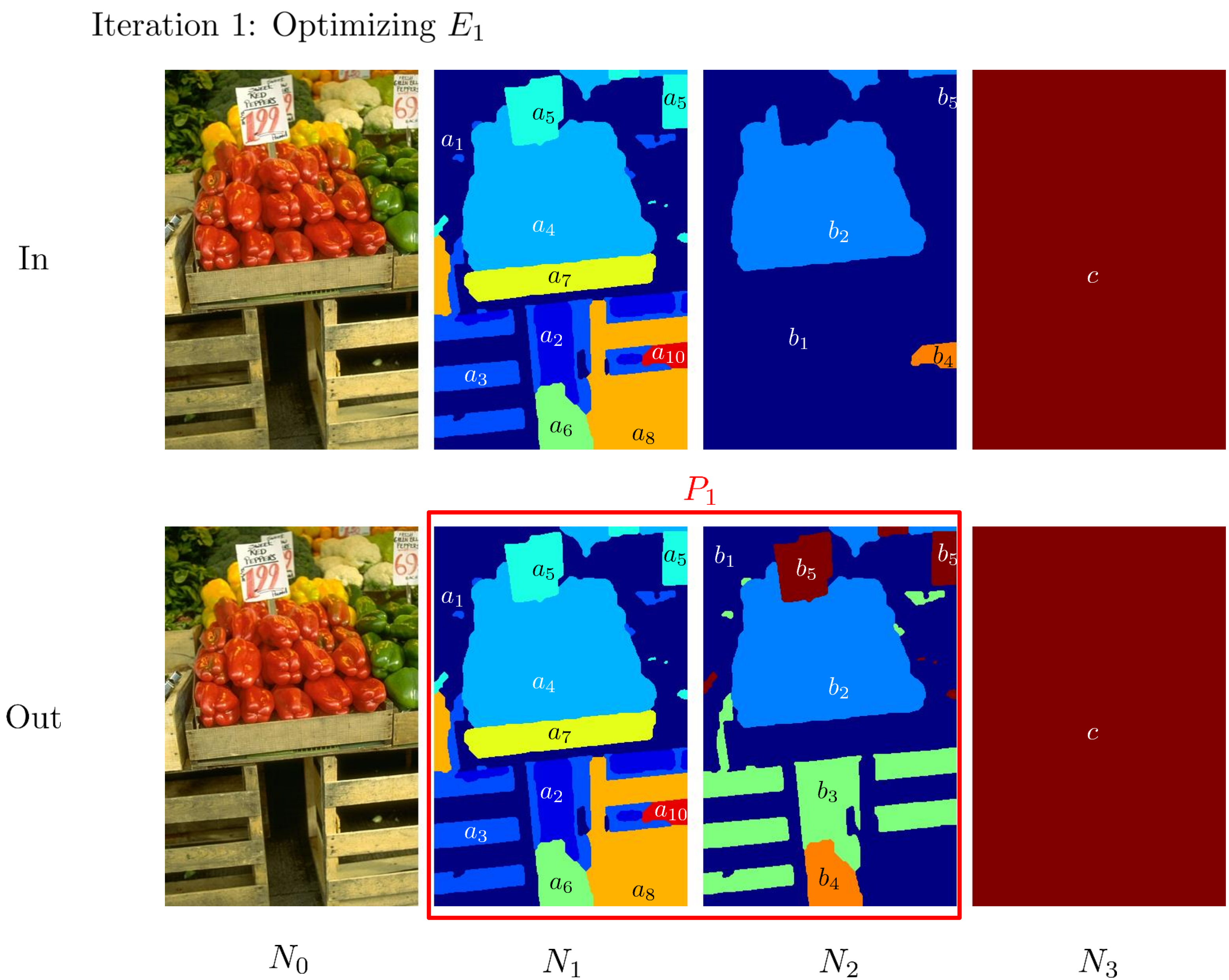
Inputs: Hierarchy 4 Levels



Iterate until convergence:

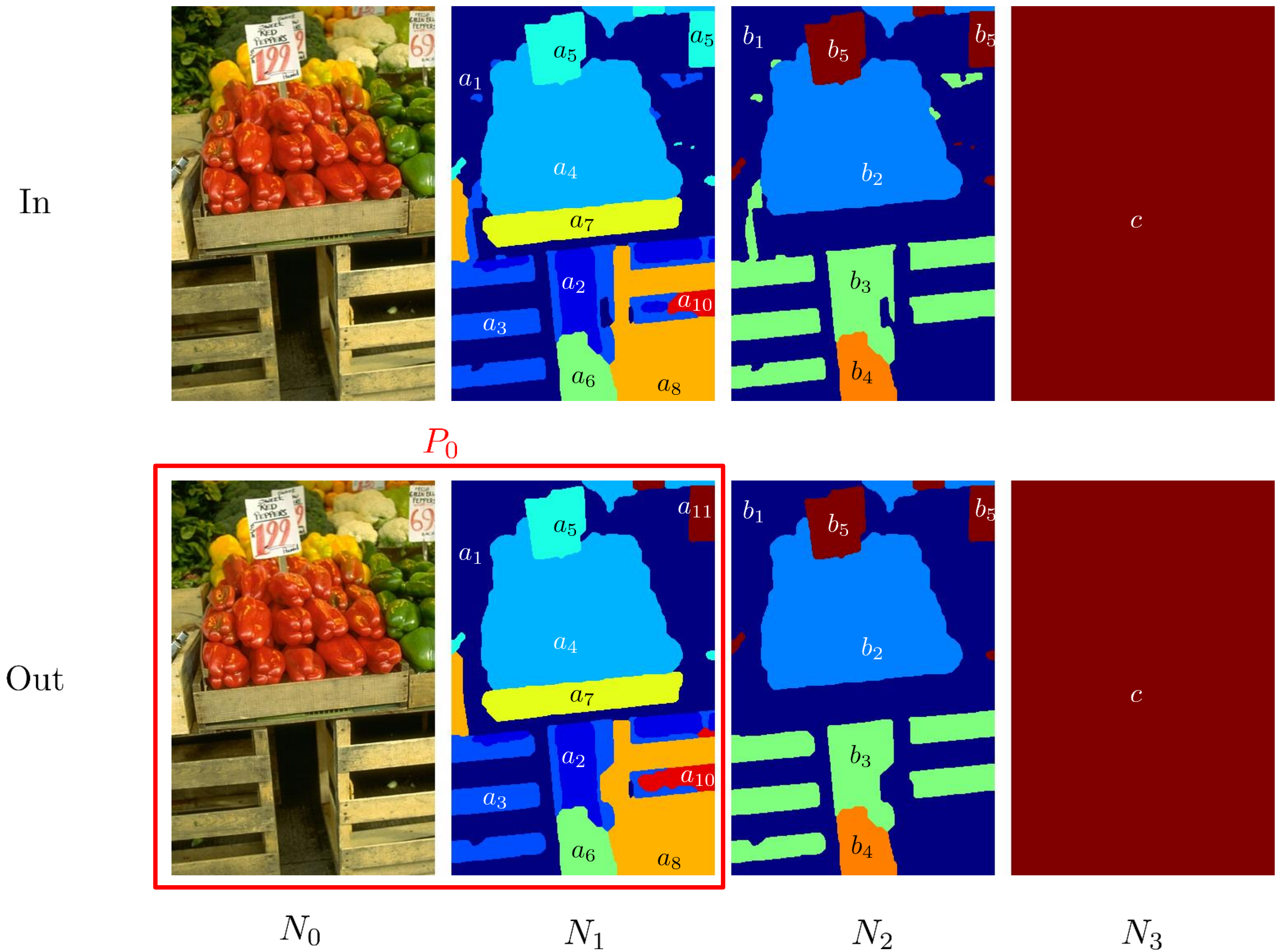
1. Optimizing E_0(α-Expansion, EM)
2. Optimizing E_1(α-Expansion, EM)
3. Optimizing E_2 (Trivial solution)

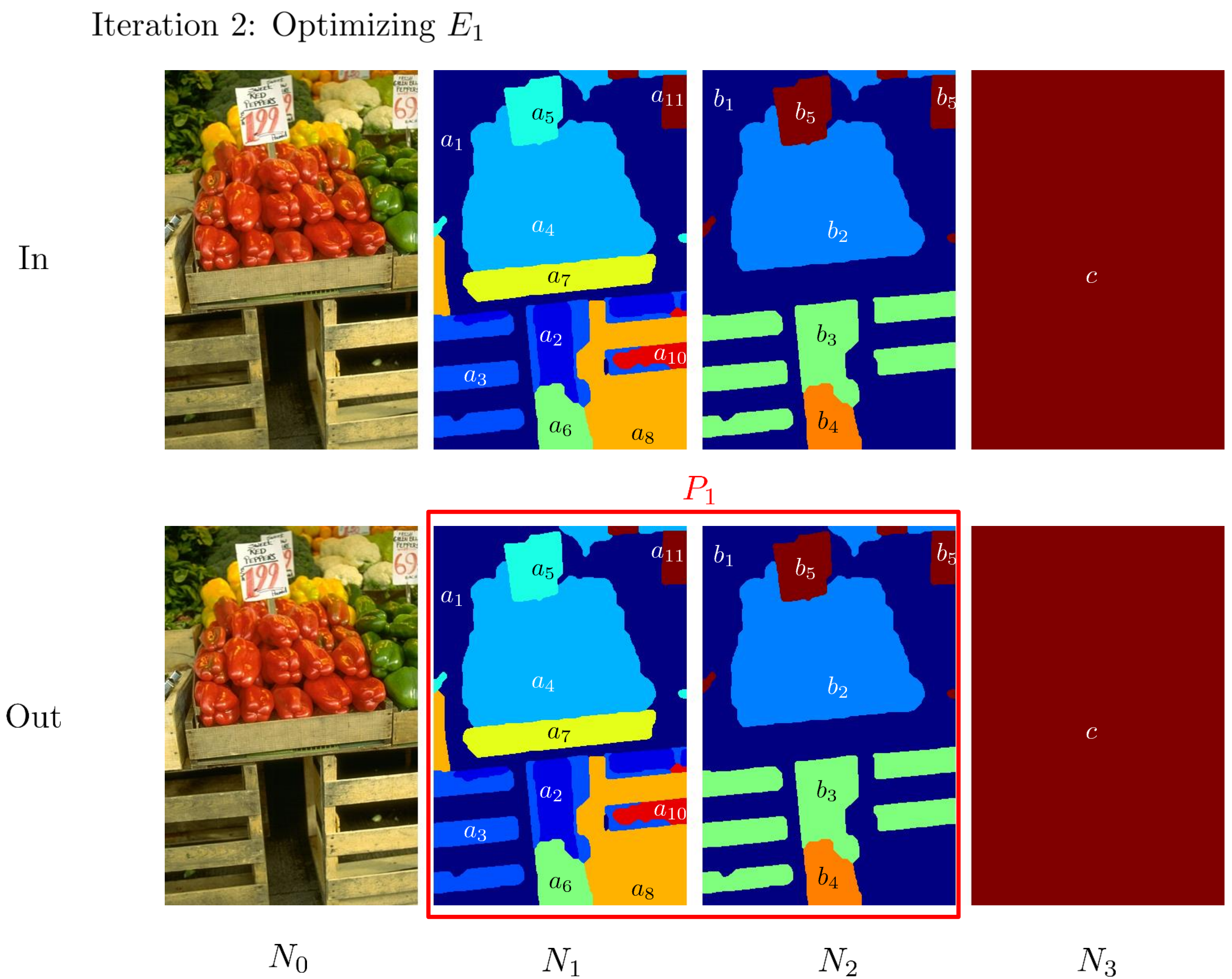

Iteration 1: Optimizing E_0
In
Out
P_0
a_1
a_2
a_3
a_4
a_5
a_6
a_7
a_8
a_9
a_{10}
a_{11}
b_1
b_2
b_3
b_4
b_5
c
N_0
N_1
N_2
N_3

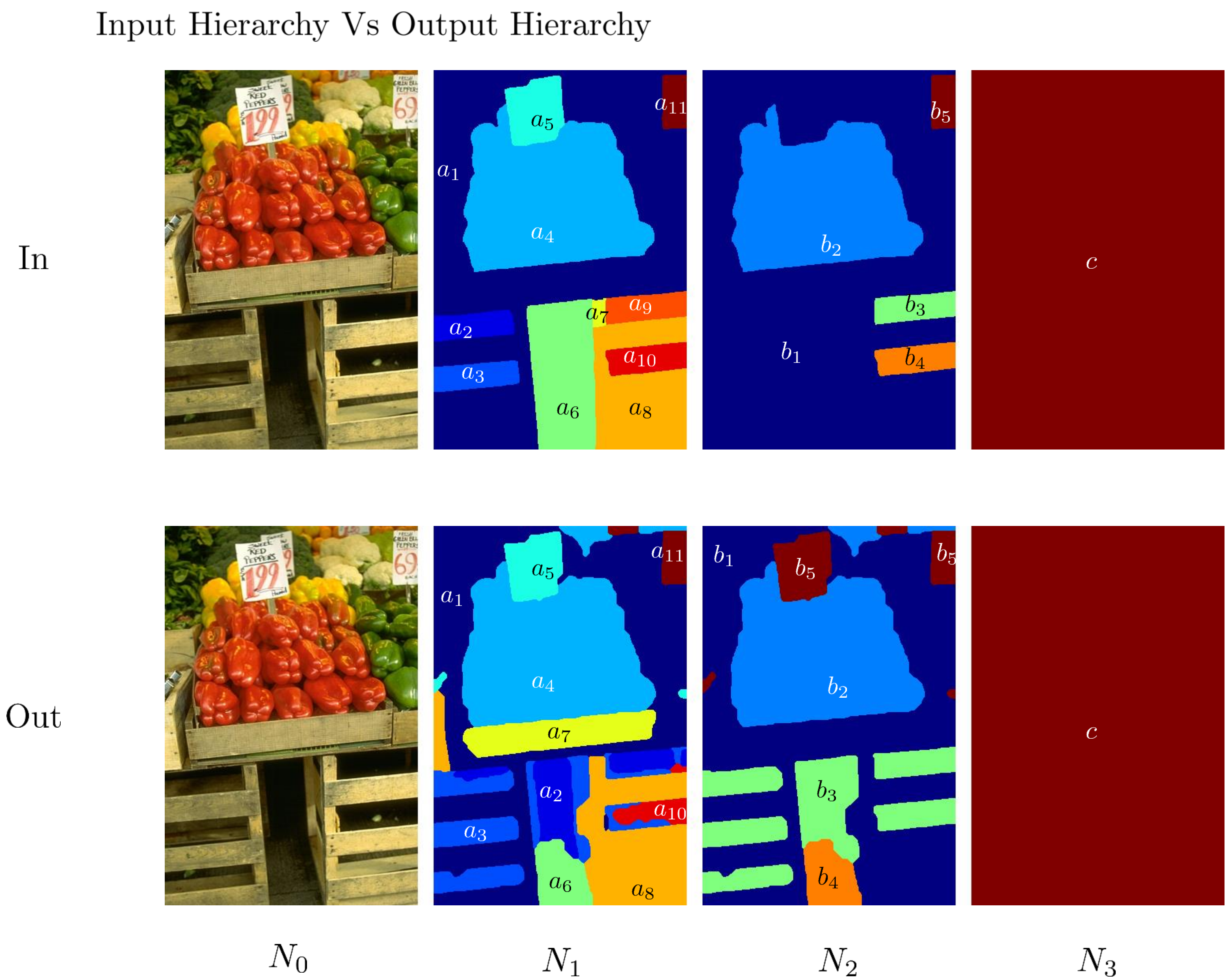


Iteration 1: Optimizing E_1
In
Out
P_1
a_1
a_2
a_3
a_4
a_5
a_6
a_7
a_8
a_{10}
b_1
b_2
b_3
b_4
b_5
c
N_0
N_1
N_2
N_3

Iteration 2: Optimizing E_0

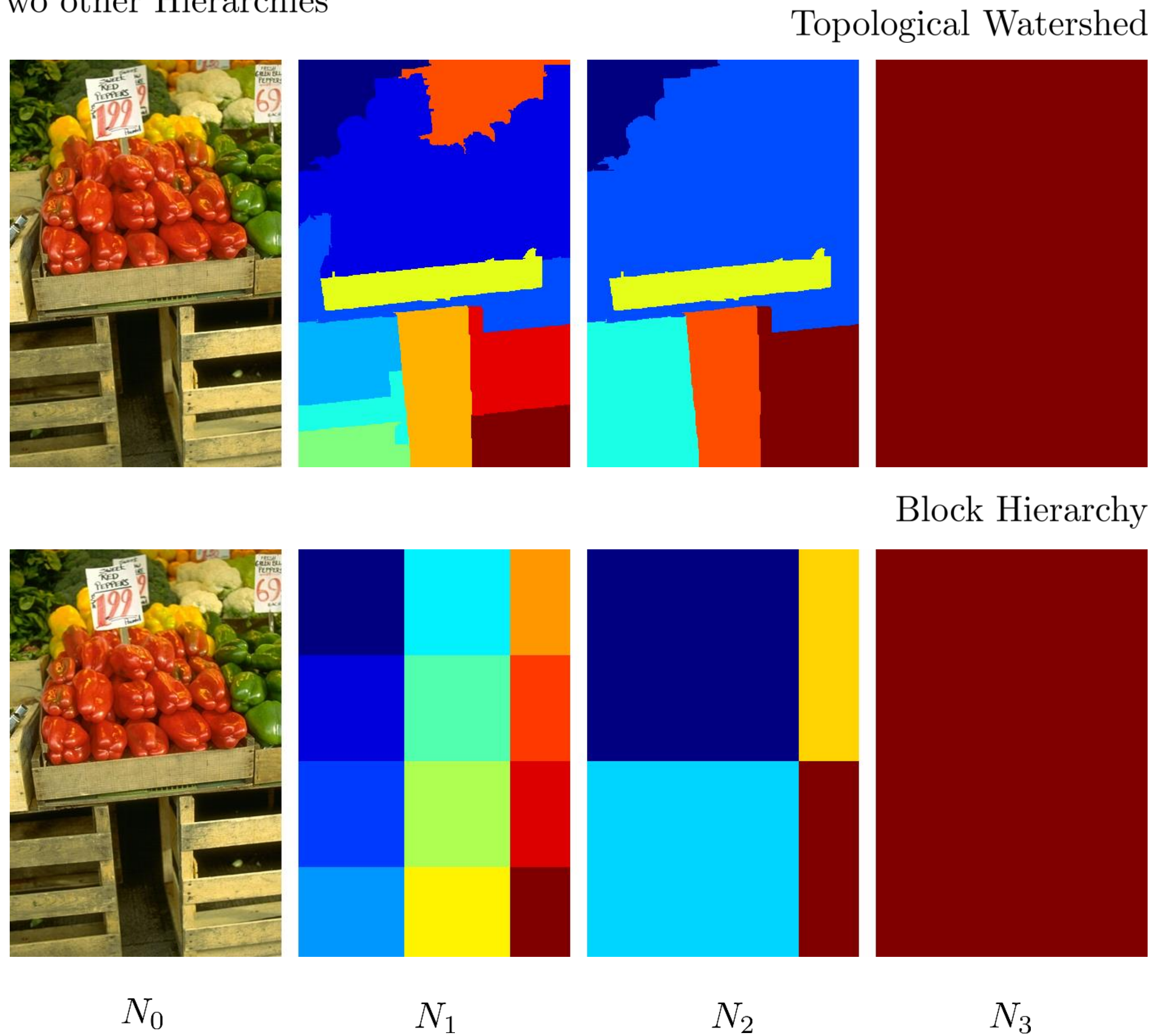


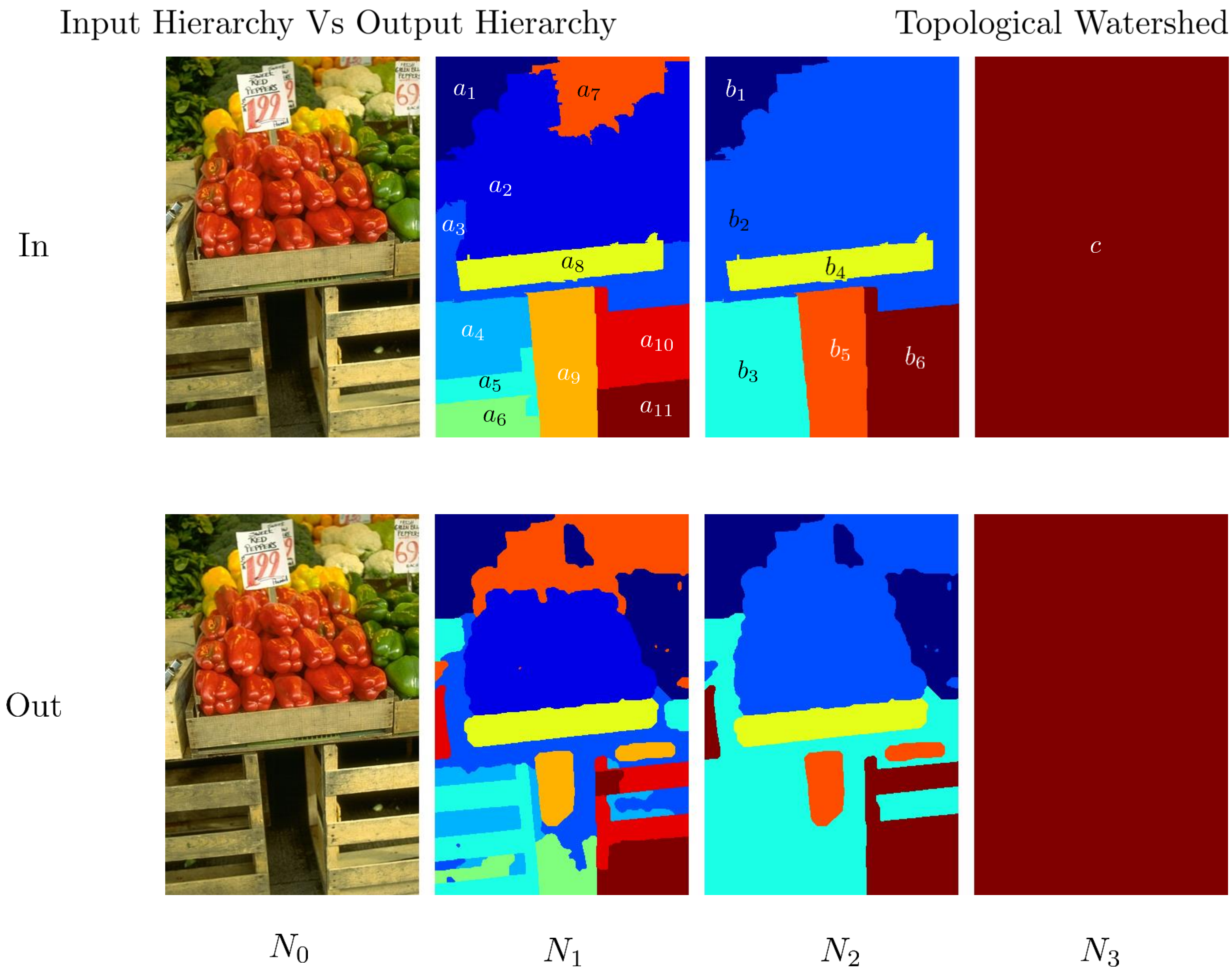

Iteration 2: Optimizing E_1
In
Out
a_1
a_2
a_3
a_4
a_5
a_6
a_7
a_8
a_{10}
a_{11}
b_1
b_2
b_3
b_4
b_5
c
P_1
N_0
N_1
N_2
N_3



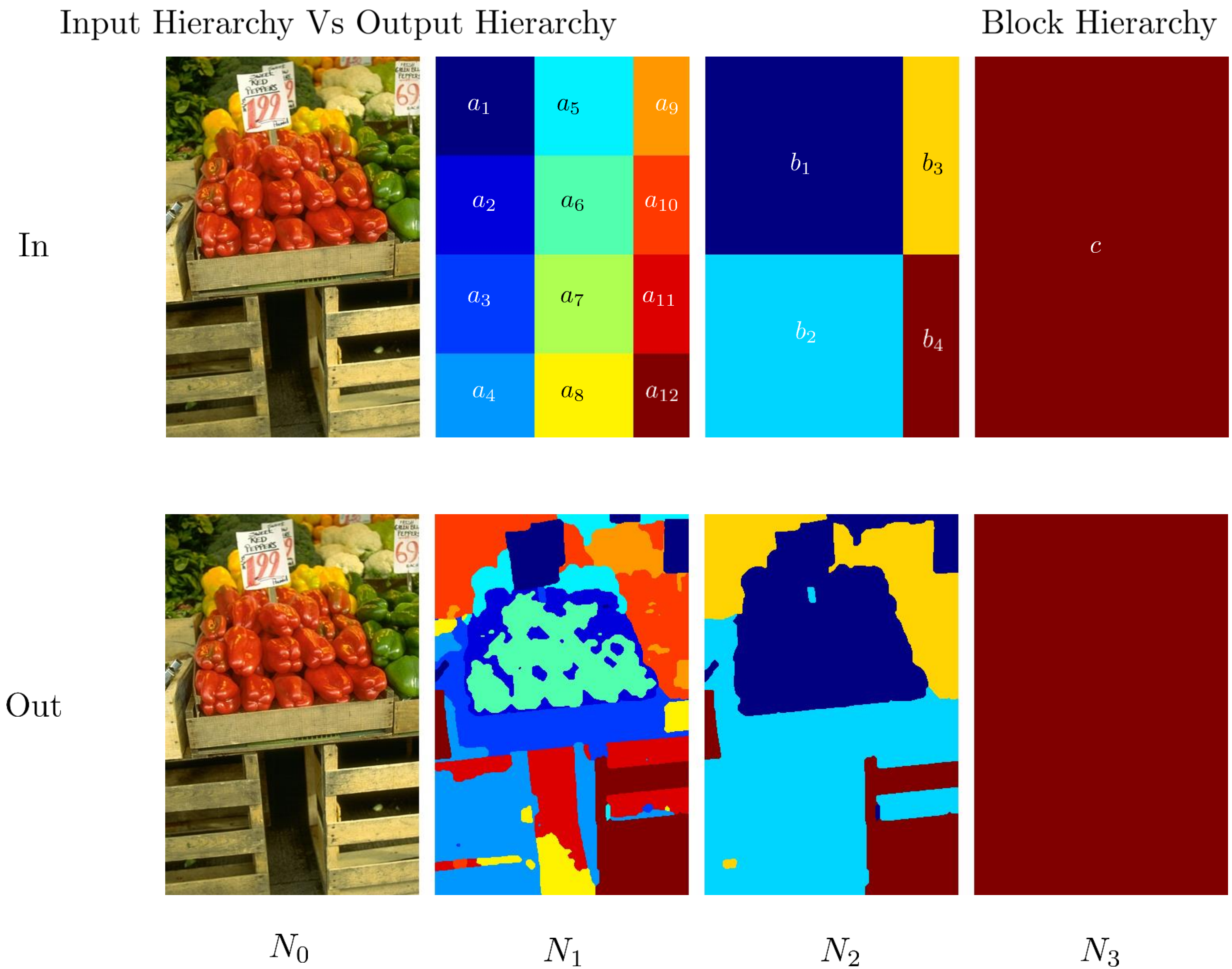
Input Hierarchy Vs Output Hierarchy
In
Out
a_1
a_2
a_3
a_4
a_5
a_6
a_7
a_8
a_9
a_{10}
a_{11}
b_1
b_2
b_3
b_4
b_5
c
N_0
N_1
N_2
N_3

Two other Hierarchies





Input Hierarchy Vs Output Hierarchy
Topological Watershed
In
Out
a_1
a_7
a_2
a_3
a_8
a_4
a_{10}
a_9
a_5
a_6
a_{11}
b_1
b_2
b_4
b_5
b_6
b_3
c
N_0
N_1
N_2
N_3


Input Hierarchy Vs Output Hierarchy
Block Hierarchy
In
Out
a_1 a_5 a_9
a_2 a_6 a_{10}
a_3 a_7 a_{11}
a_4 a_8 a_{12}
b_1 b_3
b_2 b_4
c
N_0 N_1 N_2 N_3