ESIEE PARIS

Ground truth energies for hierarchies of segmentations*

B Ravi Kiran, Jean Serra
ESIEE, Université Paris-Est

Feb 7 2012

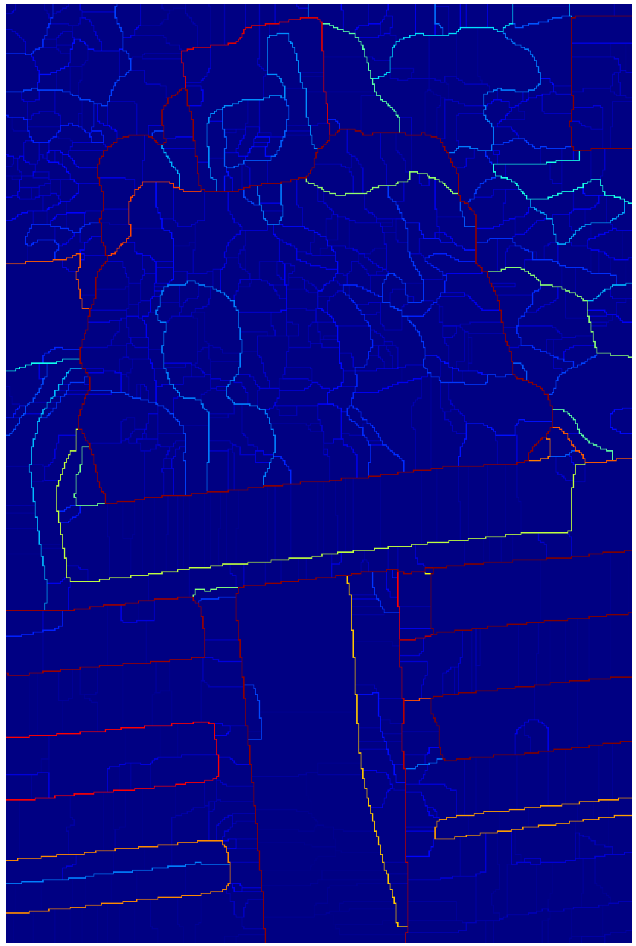
Journée ISS, Ecole des Mines

ISS

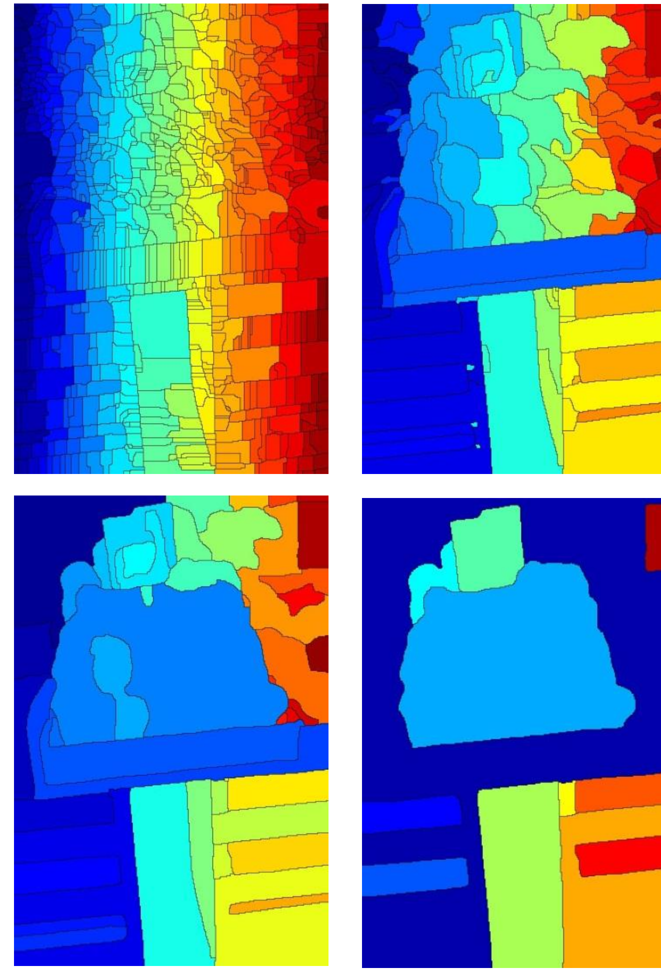
* Accepted at ISMM 2013

Hierarchy of Partitions: Example

Input Image
25098 Berkeley
Database



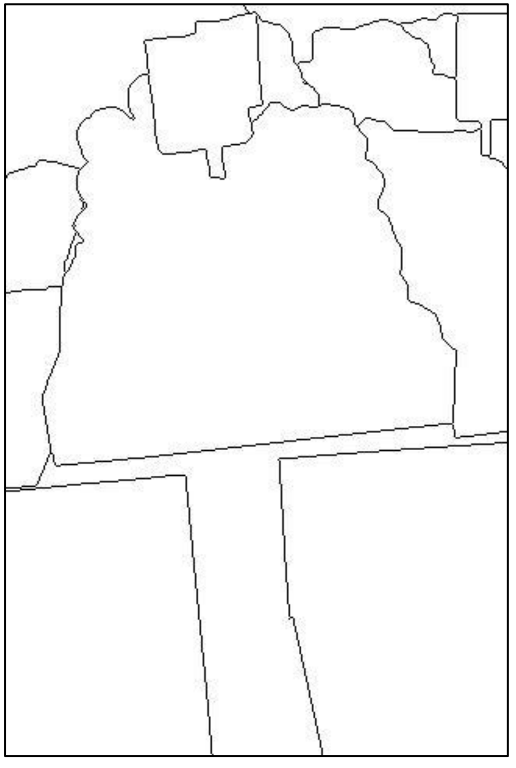
Saliency
(Ultrametric contour Map (UCM))



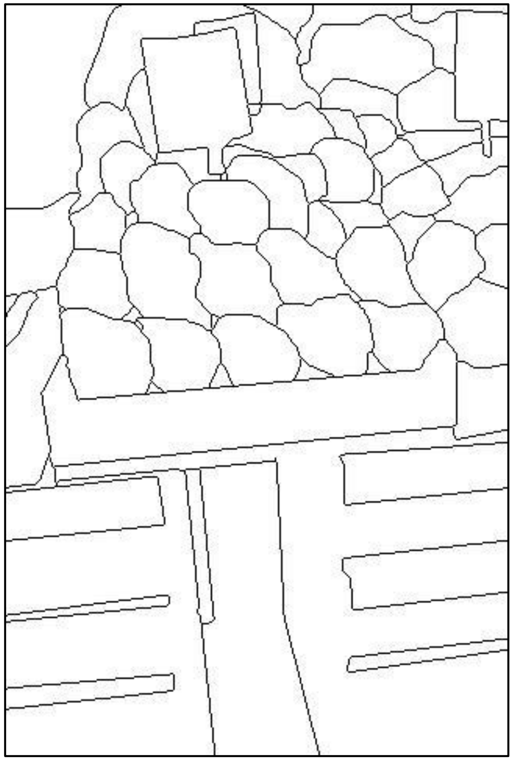
Partitions in
Hierarchy

*Single image presented for pedagogical reasons

Ground Truths



GT2



GT7

Hand drawn ground truth by multiple users/experts. No inclusion ordering is assumed here!

Problems

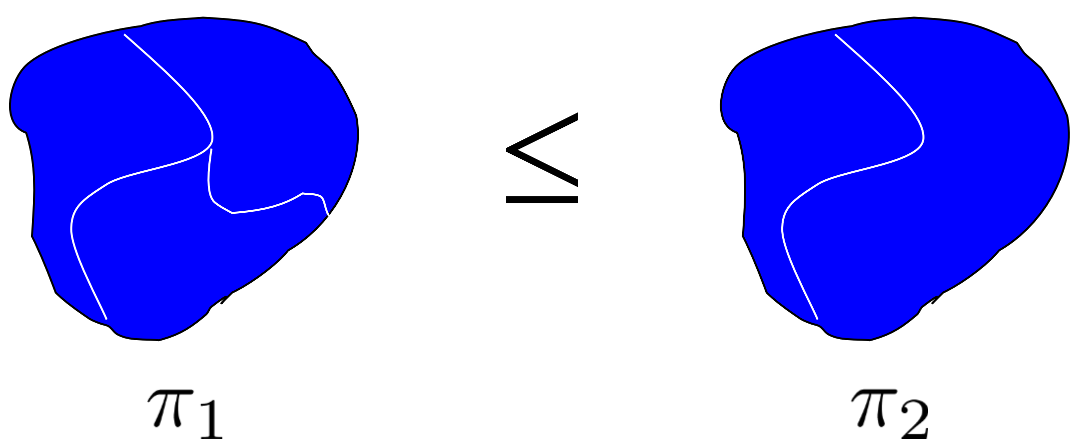
1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
    1. Closest from H -> G
    2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Hierarchy, or pyramid, of partitions

- A hierarchy of partitions is a chain of increasing partitions of some finite set E.

$$H = \{\pi_i, 0 \leq i \leq n \mid i \leq k \leq n \Rightarrow \pi_i \leq \pi_k\},$$

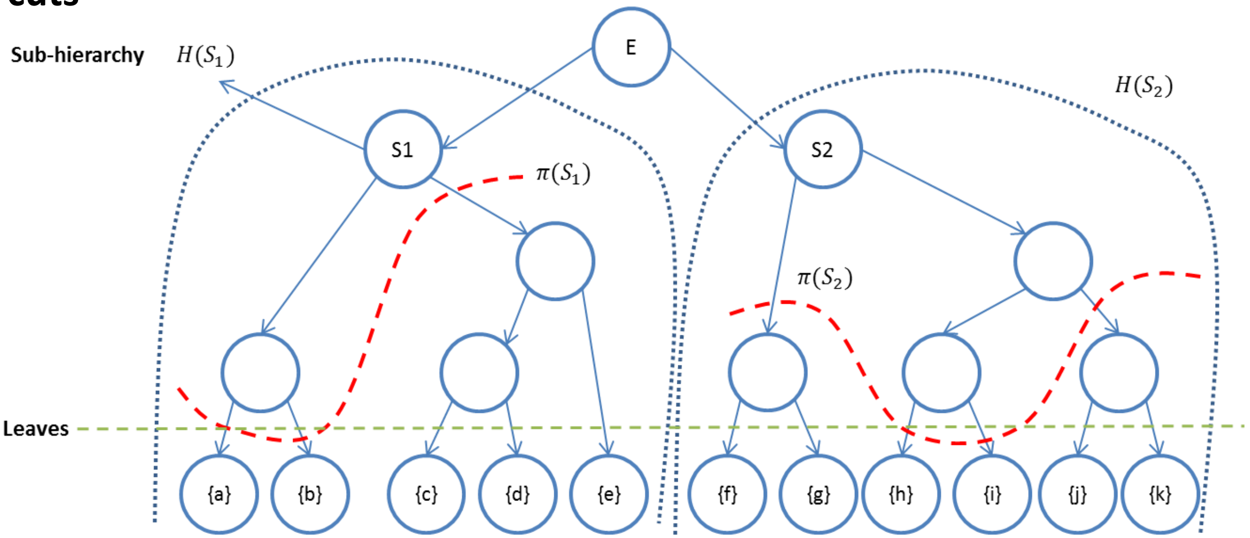
- Example of ordered partial partitions



Cuts in a hierarchy

Horizontal cuts

Non horizontal cuts



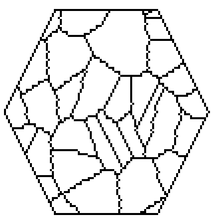
Energy minimization on Hierarchies

To minimize on hierarchies, we need three things:

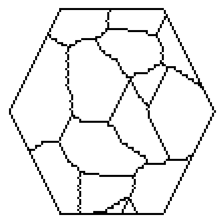
1. A *Hierarchy* *H* of partitions of E which partitions an input Image
2. A *function* f on E - Which may be the input image or another (here it will be the distance function of G)
3. An *energy* ω on f, i.e. a non-negative function over all partial partitions (i.e. partitions of any subset S of E).

h-increasingness

$\pi_1(S)$

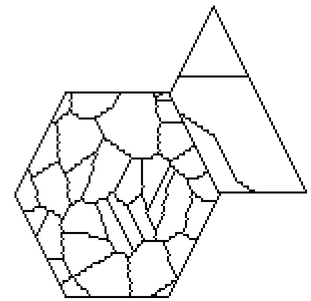


$\pi_2(S)$

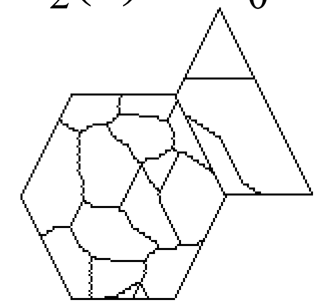


$\Rightarrow$

$\pi_1(S) \cup \pi_0$



$\pi_2(S) \cup \pi_0$



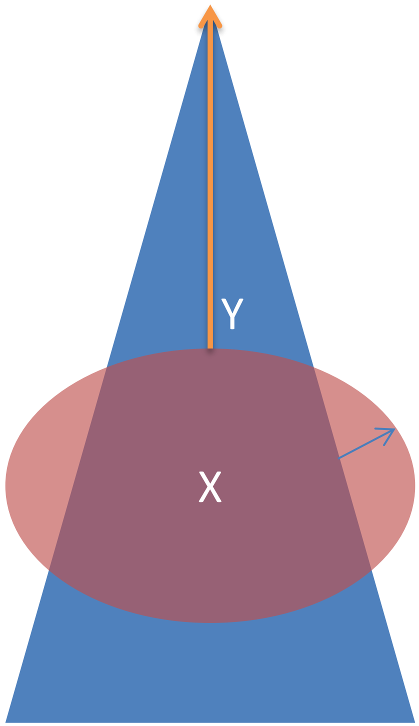
$\omega\,[\pi_1(S)] \leq \omega\,[\pi_2(S)]$ $\qquad$ $\omega\,[\,\pi_1(S) \cup \pi_0] \leq \omega\,[\pi_2(S) \cup \pi_0]$

- *Theorem* : *When the energy is h-increasing and singular, then the optimal cut at node S is either:*
  - *S itself or*
  - *the union of the optimal cuts of the sons of S*
- This gives a dynamic program which runs in linear time.

Hausdorff distance and associated problems

$$d_H(X,Y) = \max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\right\}$$

i.e. smallest disc dilation of X that contains Y and of X to contain Y



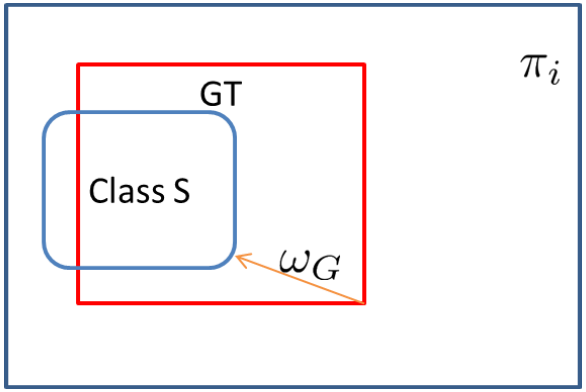
- Global Measure
- Large variations when object are asymmetric w.r.t each other

Local Hausdorff distances

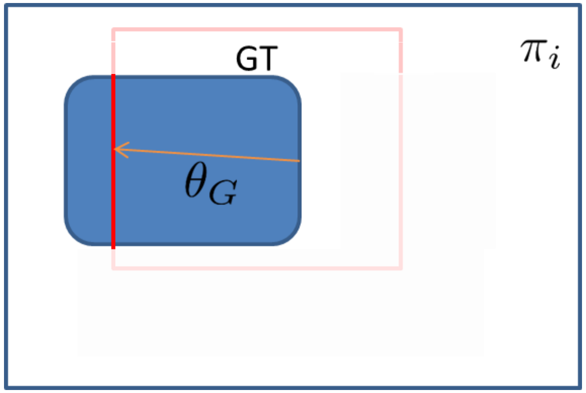
- *Local measures*: Each class S in H is assigned 2 radii: ω_G , θ_G

- Both are h-increasing energies

- Local optimization to obtain a globally optimal solution

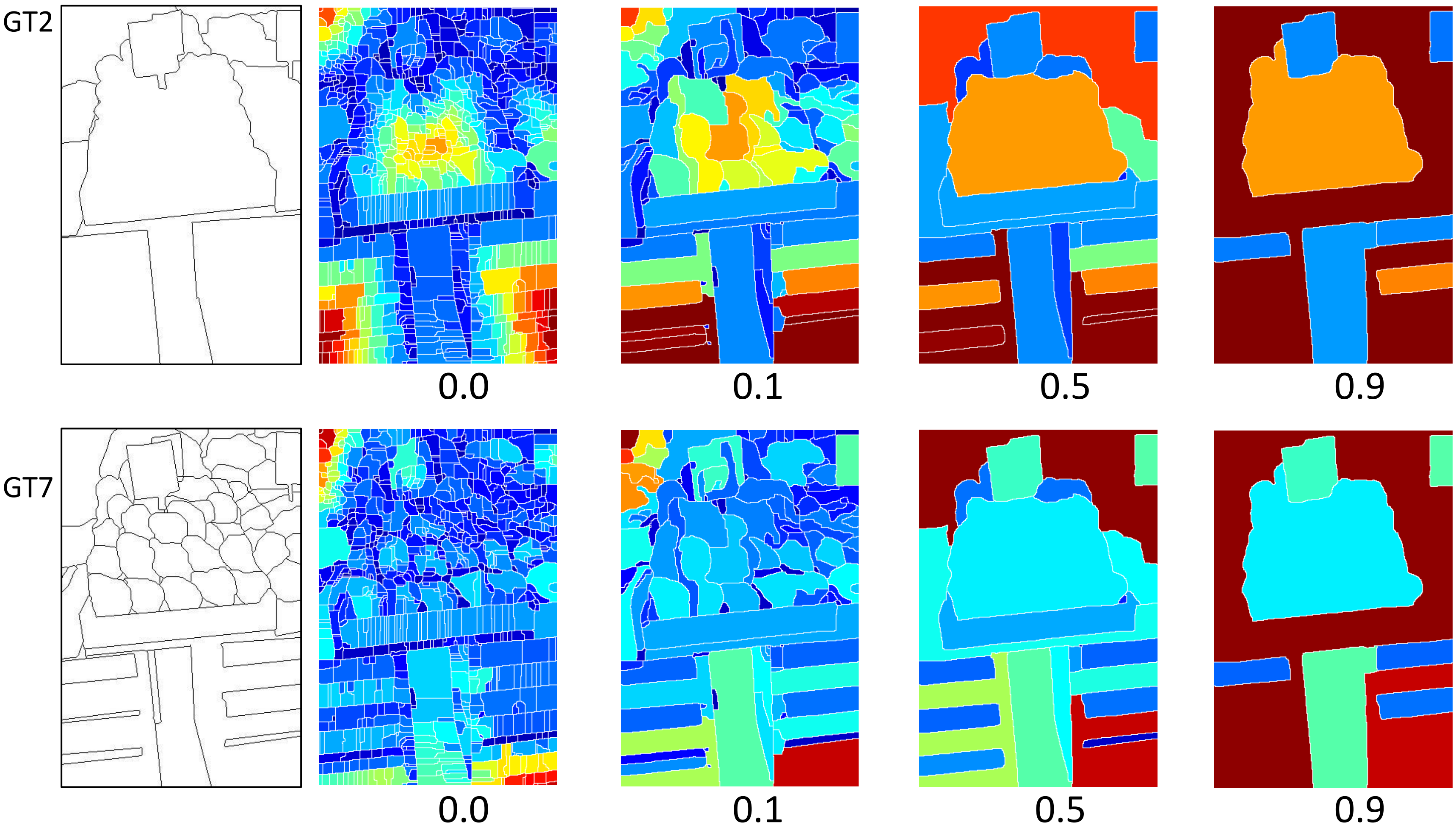


minimum radius of dilation of ground truth contour that covers the contour of S.

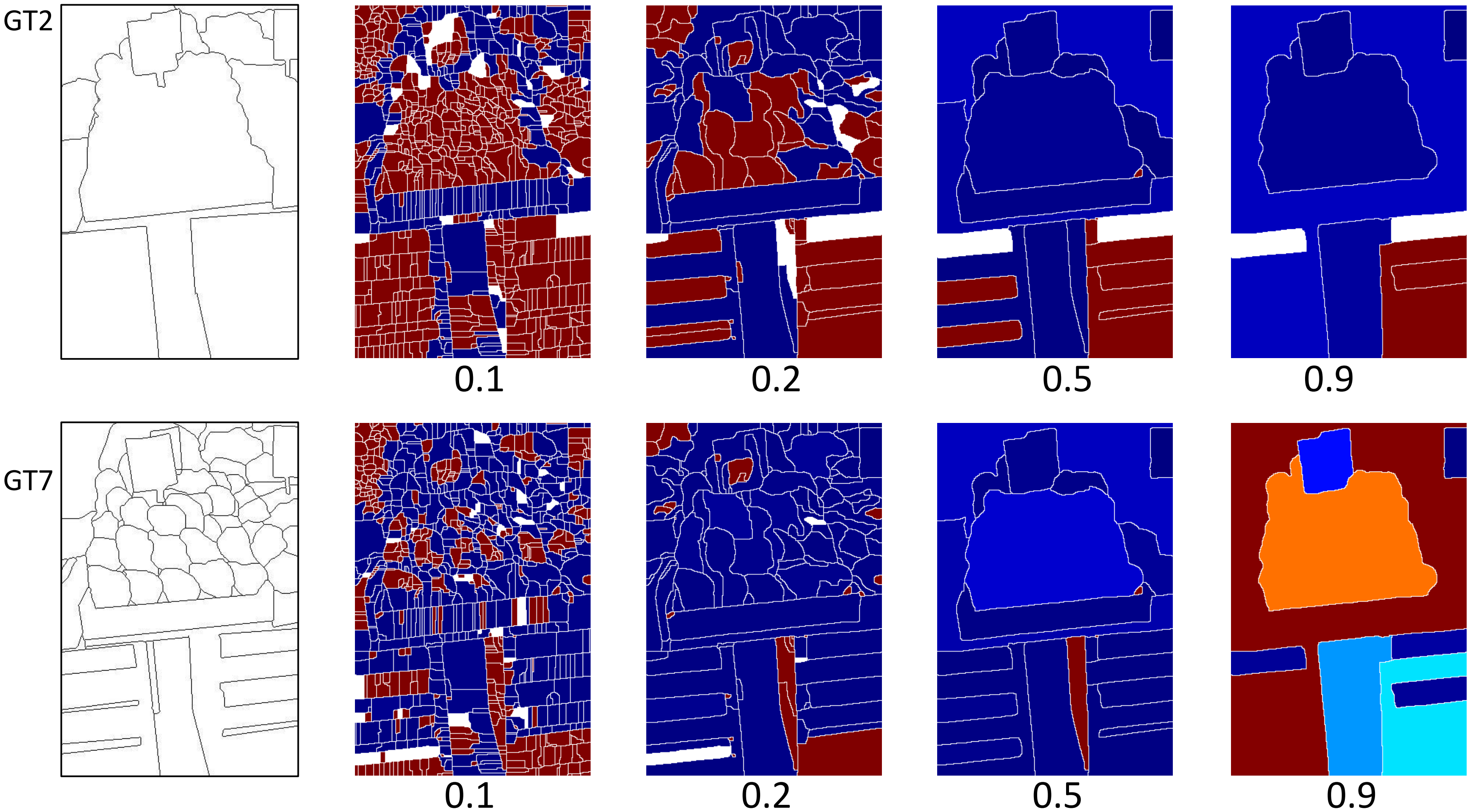


minimum radius of dilation of the contour of S to cover GT within S.

ω_{G} Energy at various levels of H



θ_{G} Energy at various levels of H

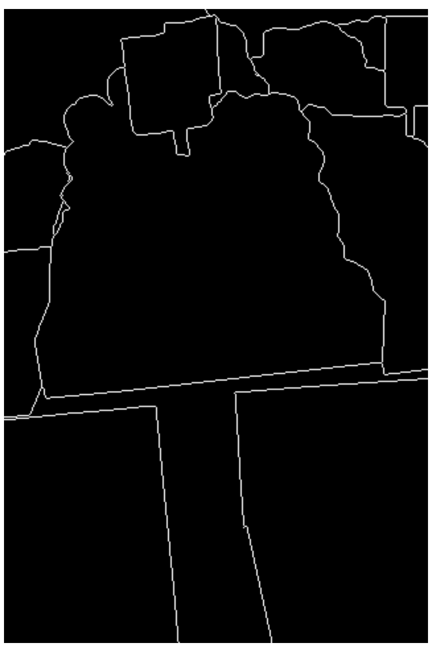


Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
   1. Closest from H -> G
   2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Optimal Cuts

Initial Image



GT2

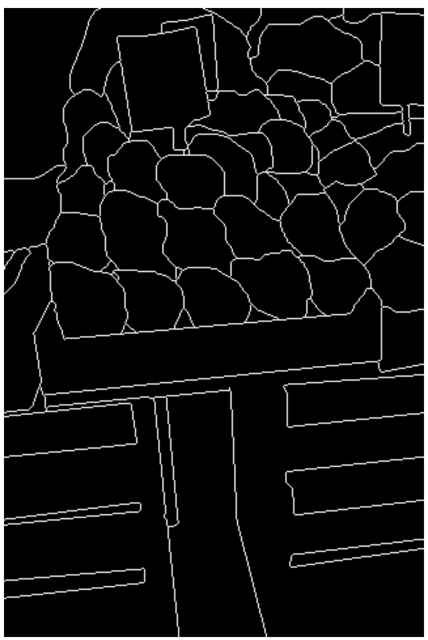
ω_{GT2}

θ_{GT2}

$\omega_{\mathrm{GT2}} + \theta_{\mathrm{GT2}}$

Optimal Cuts

Initial Image



GT7

ω_{GT7}

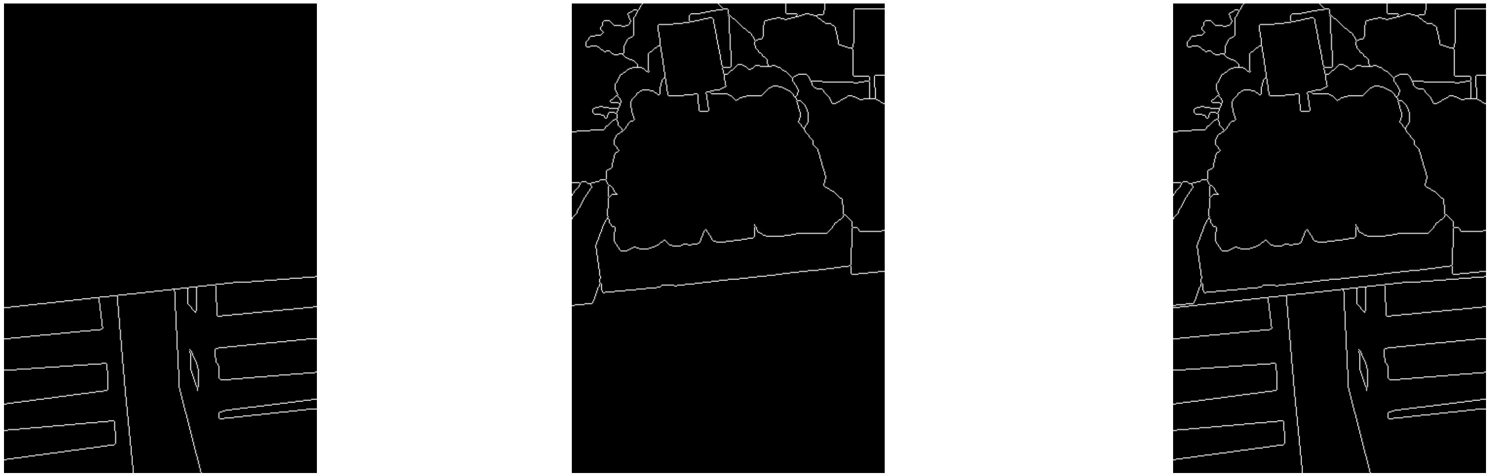
θ_{GT7}

$\omega_{\mathrm{GT7}} + \theta_{\mathrm{GT7}}$

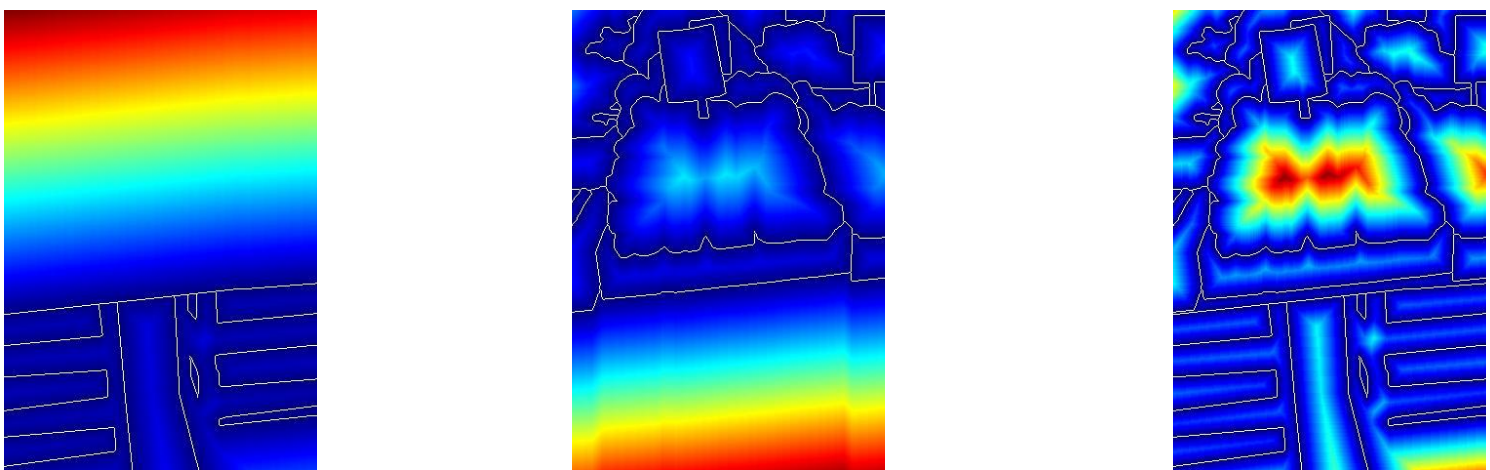
Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
   1. Closest from H -> G
   2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Composition of ground truths:



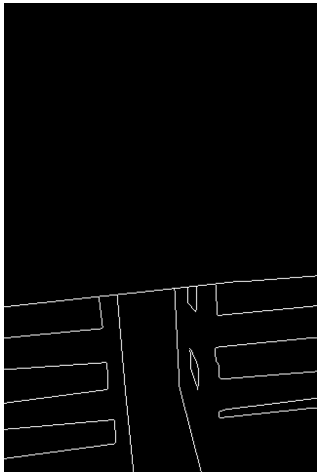
GT5_1 GT5_2 GT5 = GT5_1 + GT5_2



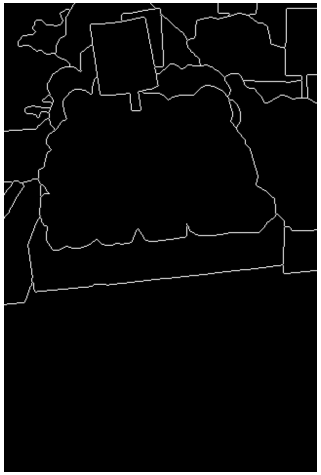
g_1 g_2 $\inf(g_1, g_2)$

The distance function of the union(sum) is the inf of the distance functions

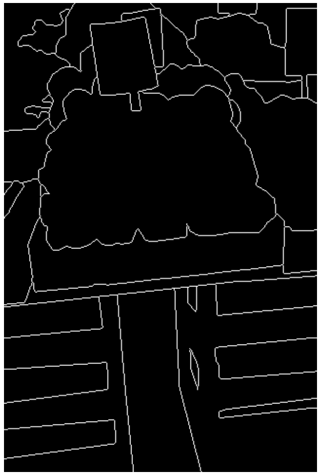
Composition of ground truths:



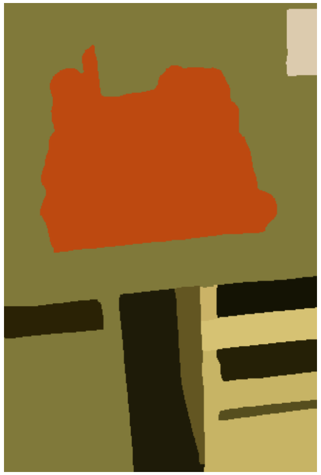
GT5_1



GT5_2



GT5 = GT5_1 + GT5_2



$\omega_{\mathrm{GT5\_1}} + \theta_{\mathrm{GT5\_1}}$

$\omega_{\mathrm{GT5\_2}} + \theta_{\mathrm{GT5\_2}}$

$\inf(\omega_{\mathrm{GT5\_1}} + \theta_{\mathrm{GT5\_1}}, \omega_{\mathrm{GT5\_2}} + \theta_{\mathrm{GT5\_2}})$

Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
    1. Closest from H -> G
    2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Global Precision-Recall Energies

The two half distances yield two local and then two global energies:

- Precision (P) : How close is on average the ground truth to the class (G->S)
- Recall (R) : How close is on average the Class contour to the Ground truth (S->G)

$$\widetilde{\omega}_G(S) = \frac{1}{\partial S} \int_{\partial S} g(x) dx$$

$$\widetilde{\theta}_G(S) = \frac{1}{G \cap S} \int_{G \cap S} g(x, \partial S) dx$$

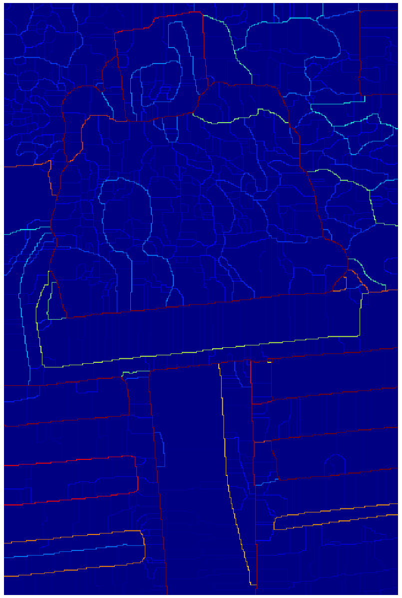
Local dissimilarity measure

$$P = \sum_{i=0}^{1} \frac{i}{N} \frac{\int_{x \in \epsilon(S_i)} (1 - g(x)).S_i(x) \mathrm{d}x}{|S_i|}$$

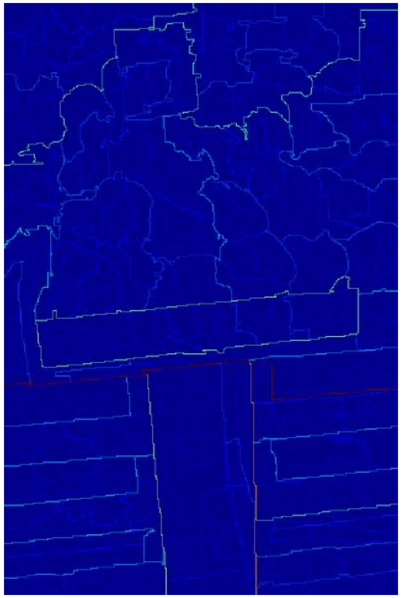
$$R = \sum_{i=0}^{1} \frac{i}{N} \frac{\int_{x \in G} (1 - g_{S_i}(x)) \mathrm{d}x}{|G|}$$

Counterpart Global similarity measures

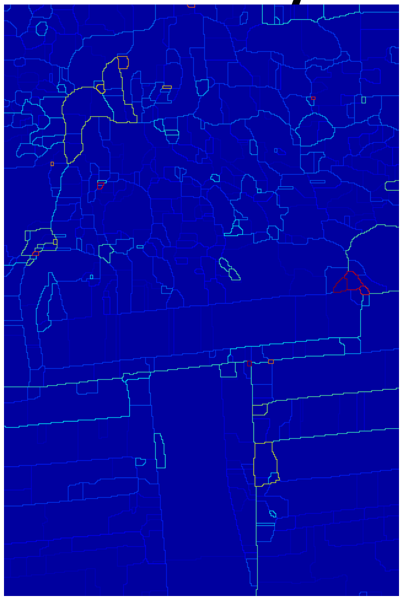
Comparing Hierarchies (saliencies)
with Precision-recall similarity measures



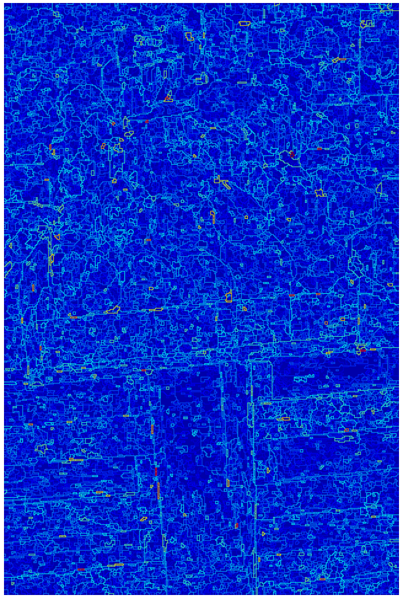
UCM



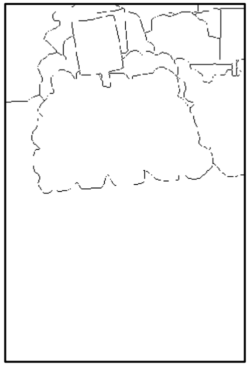
Cousty
(floodings of watershed)



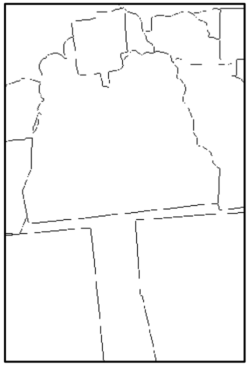
UCM random hierarchy



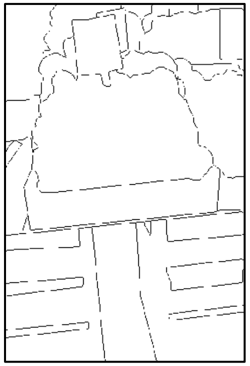
Cousty random hierarchy



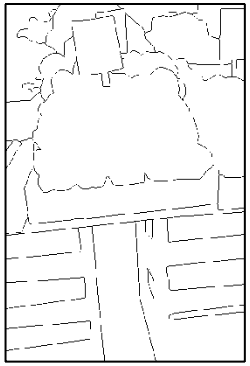
GT1



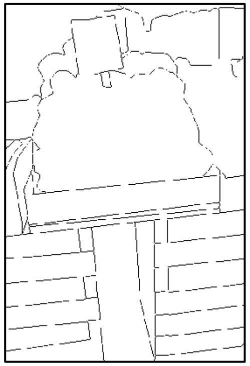
GT2



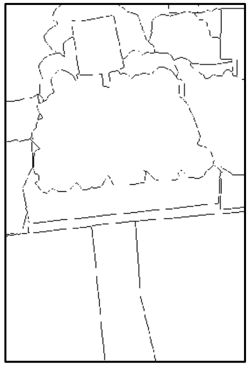
GT3



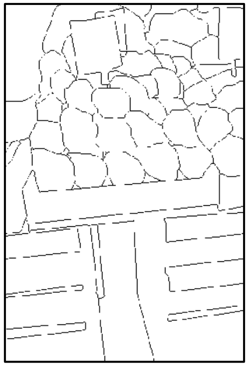
GT4



GT5



GT6



GT7

Comparing Hierarchies (saliencies)
with Precision-recall similarity measures

Image 25098	UCM	UCM random	Cousty	Cousty random
Precision energy	4.4	0.27	0.13	0.09
Recall energy	3.9	0.28	0.16	0.10

Integrals from PR equations expressed per 1000 pixels in the image

Conclusions

- h-increasing energies based on local Hausdorff distances
- Composition laws for fusing multiple ground truths
- Evaluating hierarchies against ground truth by global similarity measures

- Merci beaucoup pour
  - votre patience ☺
  - et votre attention

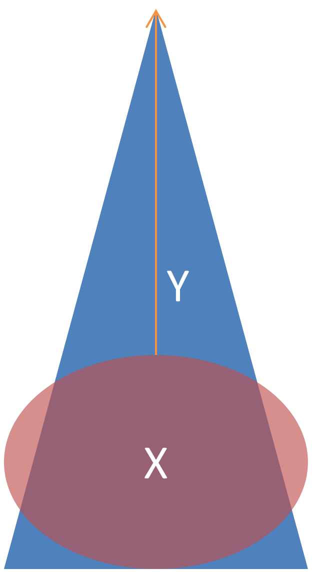
- Merci beaucoup pour
  - votre patience ☺
  - et votre attention

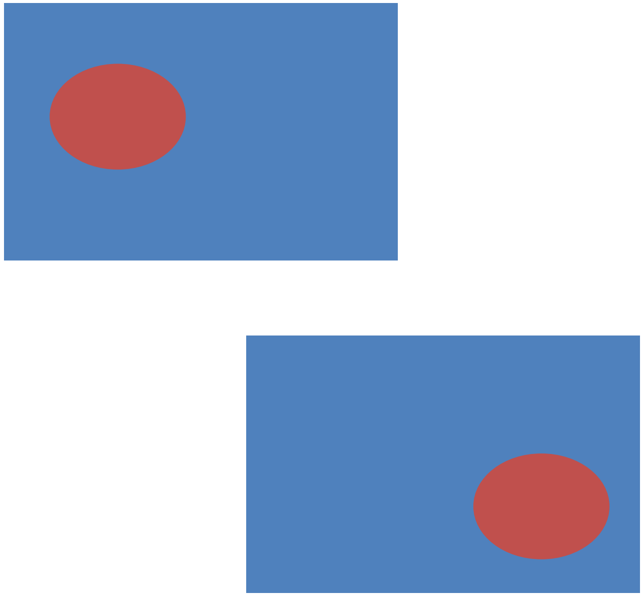
- Avez vous des questions ?

Haussdorf distance and associated problems

$$\mathrm{d}_H(X,Y) = \max\left\{\sup_{x\in X}\inf_{y\in Y}\mathrm{d}(x,y), \sup_{y\in Y}\inf_{x\in X}\mathrm{d}(x,y)\right\}$$

The smallest disc dilation of X that contains Y and of X to contain Y





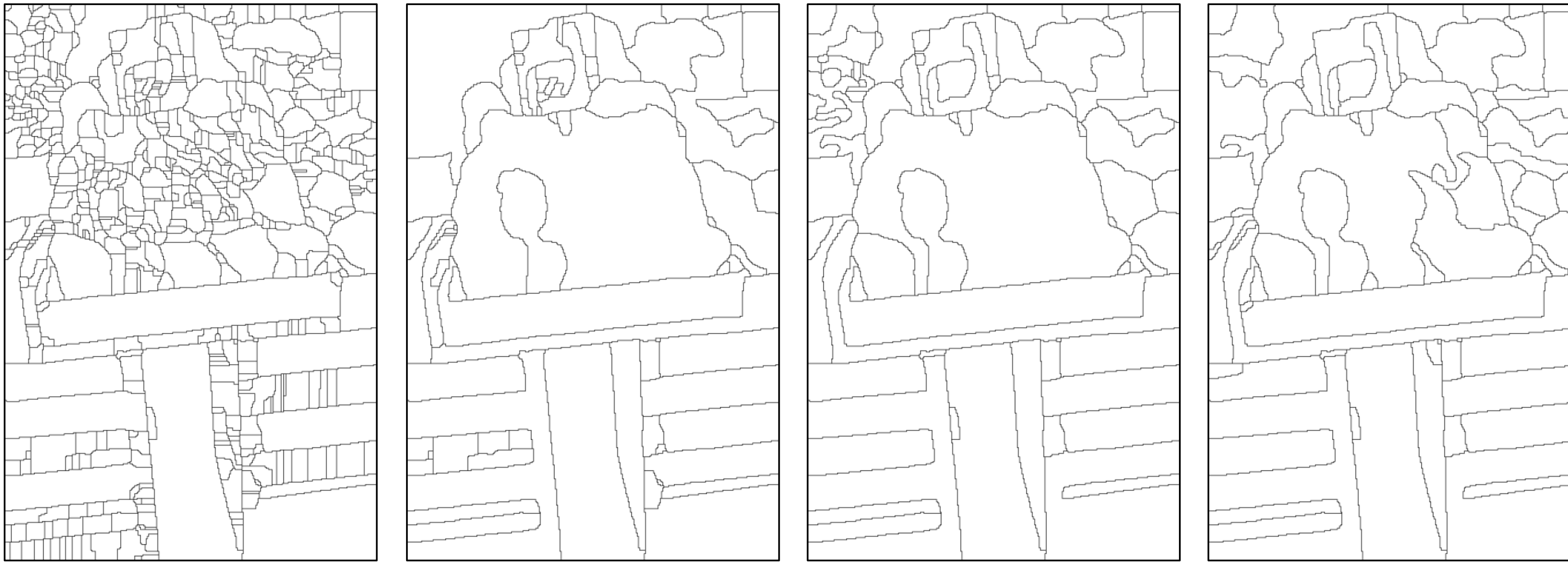
Region intersection-union based measures insensitive to topology

References

- Pablo Arbelaez, , Michael Maire, Charless Fowlkes, Jitendra Malik, Contour Detection and Hierarchical Image Segmentation PAMI 2011
- Jordi Pont-Tuset, Ferran Marques, Supervised Assessment of Segmentation Hierarchies, ECCV 2012

Conditional Energies

$$\omega_G(\pi) = \omega_G(T_i \sqcup T_2 ... \sqcup T_k) = \inf\{\omega_G(T_i)\} + \lambda.$$



λ - parametrises the difference in energies between the parent and child partial partition

π - Partial partition

T_i - one of the sons of π

$$\pi^* = \text{Optimal Cut if it arrives first OR} \quad |\pi| = Ncomps_{GT}$$

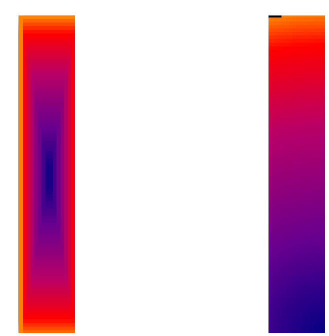
Half Haussdorf distances

$$\widetilde{\omega}_G(S) = \frac{1}{\partial S}\int_{\partial S} g(x)dx$$

$$\widetilde{\theta}_G(S) = \frac{1}{G\cap S}\int_{G\cap S} g(x,\partial S)dx$$

Another variant consists in replacing the supremum that appears in the half haussdorf relation by a L^p (p=1) sum, which gives less importance to the farthest zones.

$$\widetilde{\theta}_G(S) = \frac{1}{G\cap S}\int_{G\cap S} g(x,\partial S)dx + \mu\frac{d}{dx}g(x,\partial S)dx$$



Overview

- Problem and motivation
  - Evaluating hierarchies of segmentations
    - Hierarchy vs Ground truth
    - Hierarchy vs Ground truths
    - Hierarchies vs Ground truth
- Energy minimization on Hierarchies
  - h-increasingness
  - Optimal cuts
- Half Haussdorf energies
  - Ground truth energy (radius)
  - Class energy (radius)
  - Composition of energies
  - Conditional energies
  - Precision-Recall energies