Constrained Optimization on Hierarchies of partitions

Bangalore Ravi KIRAN
CAOR Ecoles Des Mines
16 Jan 2015
Overview

- Notations and structures
- Review on Optimization on Hierarchies
- Dynamic Programming
- Braids of Partitions
- Energetic Order and Energetic-Lattices
- Reviewing Braids
- Constrained Optimization
Braid of Partitions

A Braid $B$ with a monitoring hierarchy $H$, is a family of partitions, where the refinement supremum between two partitions in $B$ is an element of $H$.

$$\forall \pi_1, \pi_2 \in B \Rightarrow \pi_1 \vee \pi_2 \in \prod(H, E) \setminus \{E\}$$

![Diagram of family of partitions and monitor hierarchy](image)

$B_1 = \{\pi_1, \pi_2, \pi_3\} \checkmark$  \hspace{1cm} $B_2 = B_1 \cup \pi_x \times$
Generalizing Parent-Child Relations

Calculating Supremum of Partitions (components of intersection graph)
Braid: Composing hierarchies

Input Image

Watershed hierarchy (Area Attribute)

Watershed hierarchy (Volume Attribute)
Composing Hierarchies

\[ \pi \in H_1 \]

\[ \pi' \in H_2 \]

Intersection

Net opening

[Kiran-Serra PRL 2014]
Braid: Composing hierarchies

Monitor Hierarchy

Watershed hierarchy (Area Attribute)

Watershed hierarchy (Volume Attribute)
Why Braids

- Uncertain partition boundaries $\implies$ many possible partial partitions
- Multivariate segmentations and superpixel segmentations.
- Composition of hierarchical segmentations
- The dynamic program works for the family of braids, and ensures better infimum for over composition of hierarchies with non-trivial monitors.

No single GT is a refinement of the mean-shift segmentation, but their suprema are!

[Unnikrishnan et al. 2007]
Comparing Segmentations

\[ \pi_1(S) \leq \pi_2(S) \]  Refinements
\[ \pi_1(R) \leq \pi_2(R) \]

\[ \text{Braid structure} \]
\[ \text{Noise or Overlap: } T_1 \not\subset T_2 \text{ nor } T_2 \not\subset T_1 \]

Braids accommodate variability in both Expert and Algorithm segmentations
Constraint Partition Sets: Braid Structure

Cuts with $N = 5$
h-increasingness on BOP

\[ \pi_1(S') \not\leq \pi_2(S') \quad \pi_1(S') \sqcup \pi_0 \not\leq \pi_2(S') \sqcup \pi_0 \]

\[ \omega(\pi_1(S')) \leq \omega(\pi_2(S')) \implies \omega(\pi_1(S') \sqcup \pi_0) \leq \omega(\pi_2(S') \sqcup \pi_0) \]

[PhD Thesis]
Highlights

1. Braids generalize refinement relations on hierarchies

2. Monitor hierarchy might not be part of the Braid itself, but is inherent when calculating supremum of partitions.

3. Braids provide an algorithm independent partition organization structure.

4. Braids are inherent structures in tree-structured optimization problems.

5. Braids still maintain local-global structure for dynamic programs.