Constrained Optimization on Hierarchies of partitions

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Overview

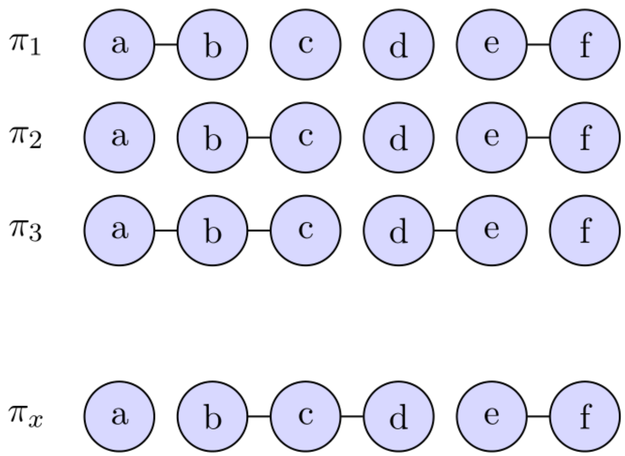
- Notations and structures
- Review on Optimization on Hierarchies
- Dynamic Programming
- Braids of Partitions
- Energetic Order and Energetic-Lattices
- **Reviewing Braids**
- **Constrained Optimization**

Braid of Partitions

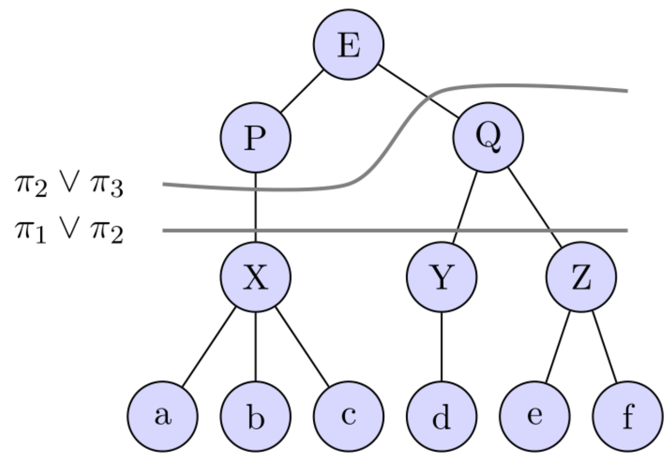
A Braid B with a monitoring hierarchy H, is a family of partitions, where the refinement supremum between two partitions in B is an element of H.

$$\forall\ \pi_1, \pi_2 \in B \quad \Rightarrow \quad \pi_1 \vee \pi_2 \in \Pi(H, E) \setminus \{E\}$$

Family of partitions

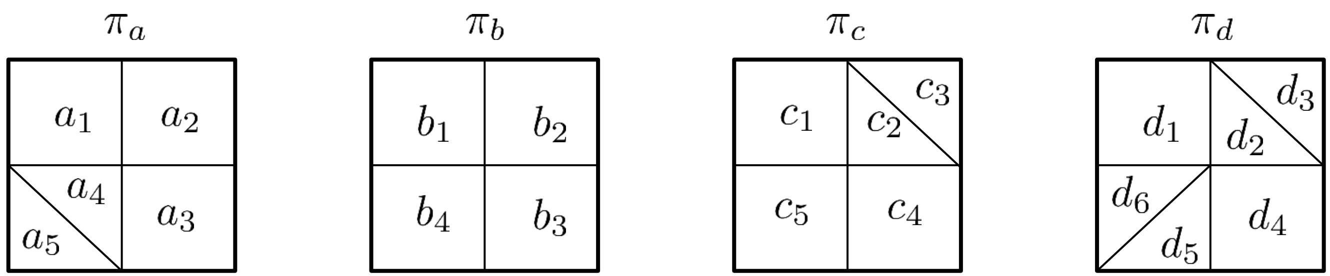


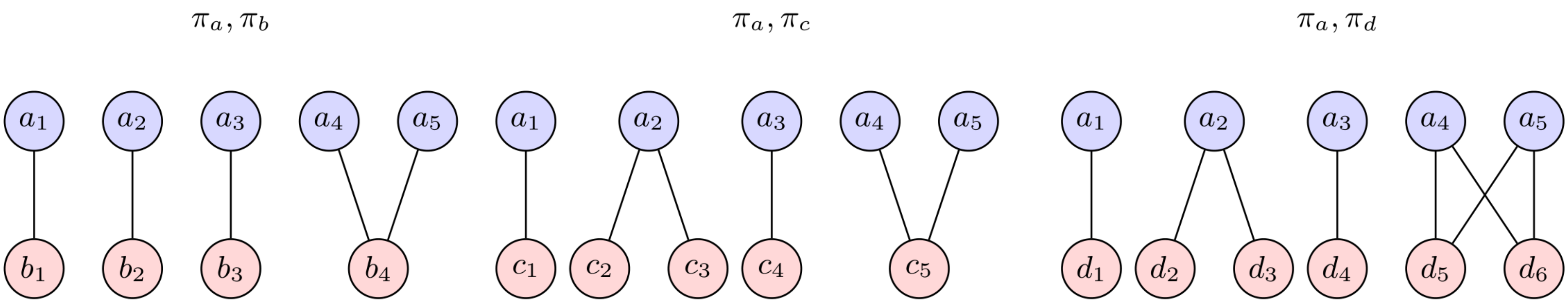
Monitor hierarchy H



$B_1 = \{\pi_1, \pi_2, \pi_3\}$ ✓ $\quad B_2 = B_1 \cup \pi_x$ ✗

Generalizing Parent-Child Relations

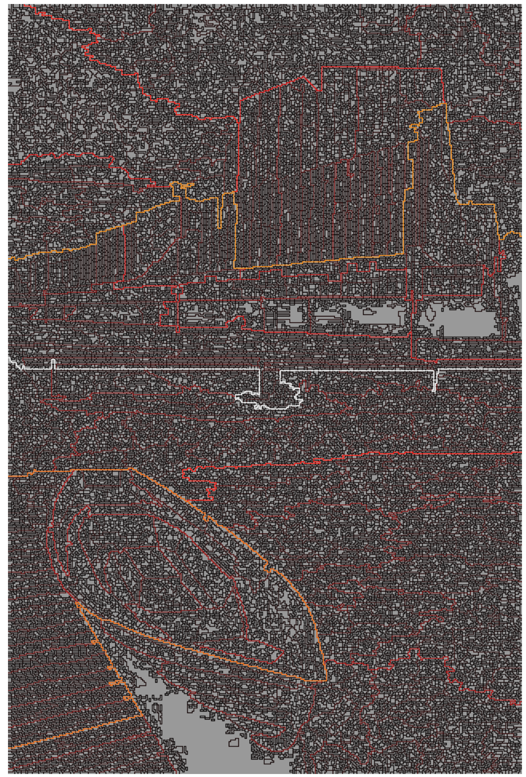




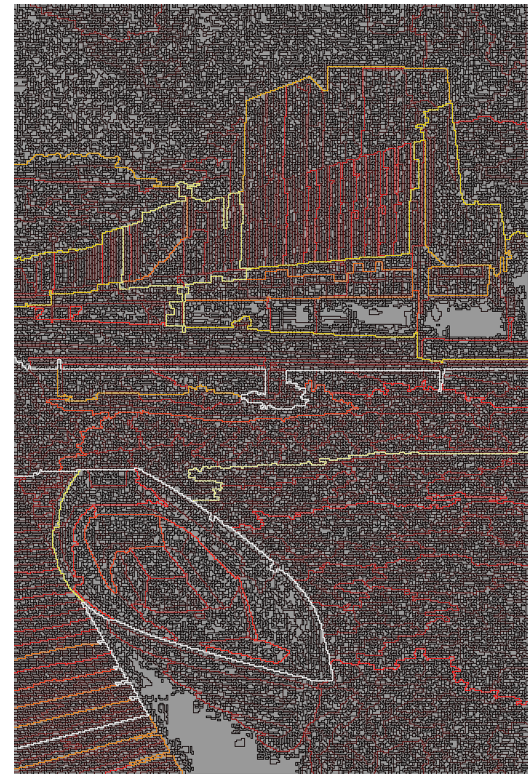
Calculating Supremum of Partitions (components of intersection graph)

Braid: Composing hierarchies

Input Image

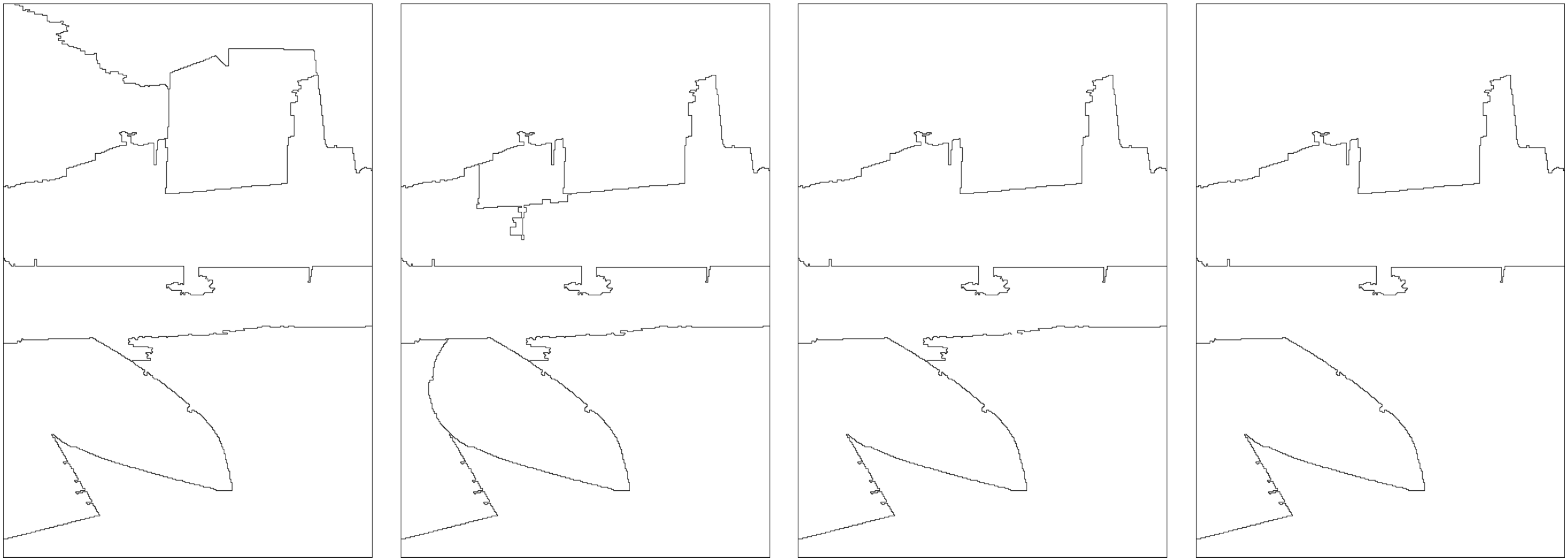


Watershed hierarchy
(Area Attribute)



Watershed hierarchy
(Volume Attribute)

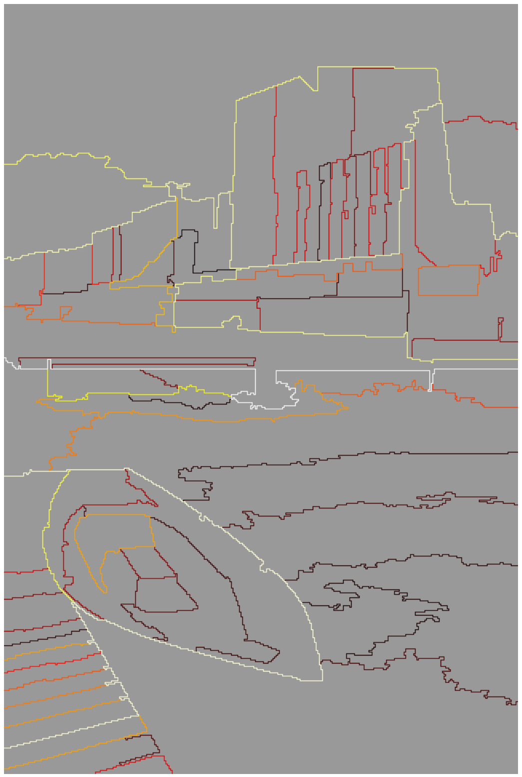
Composing Hierarchies



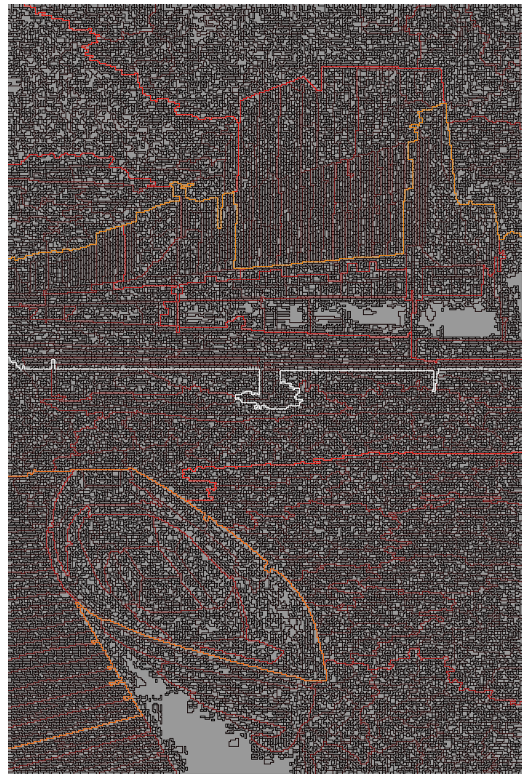
$\pi \in H_1$ | $\pi' \in H_2$ | Intersection | Net opening

[Kiran-Serra PRL 2014]

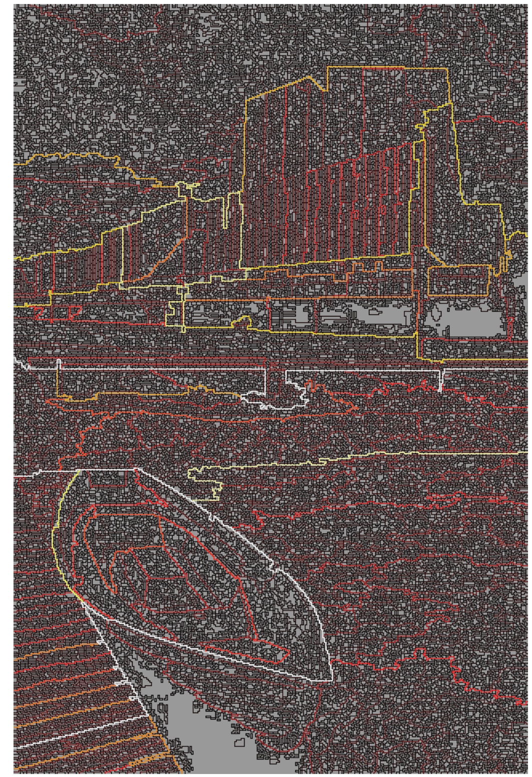
Braid: Composing hierarchies



Monitor Hierarchy



Watershed hierarchy
(Area Attribute)

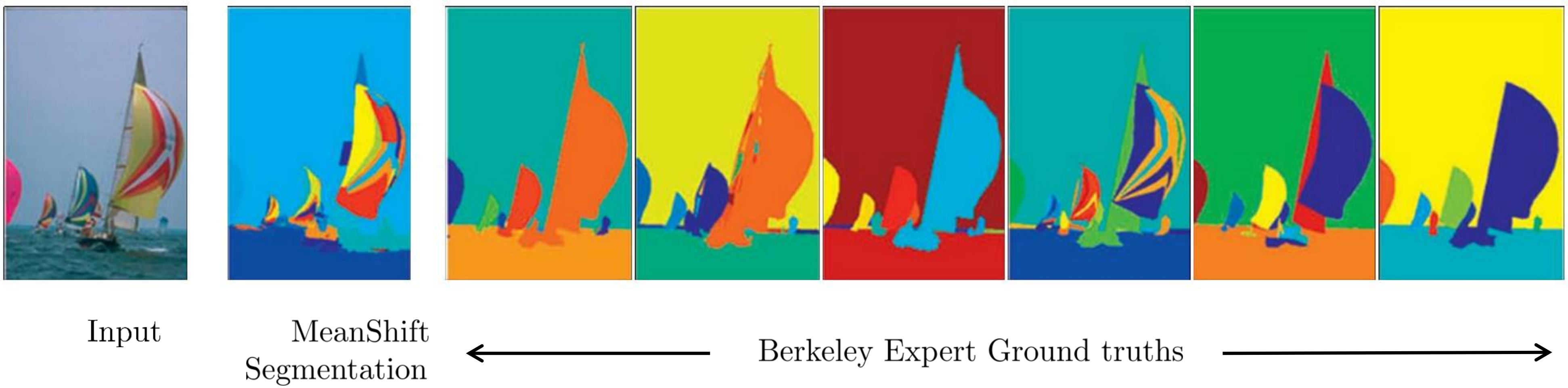


Watershed hierarchy
(Volume Attribute)

Why Braids

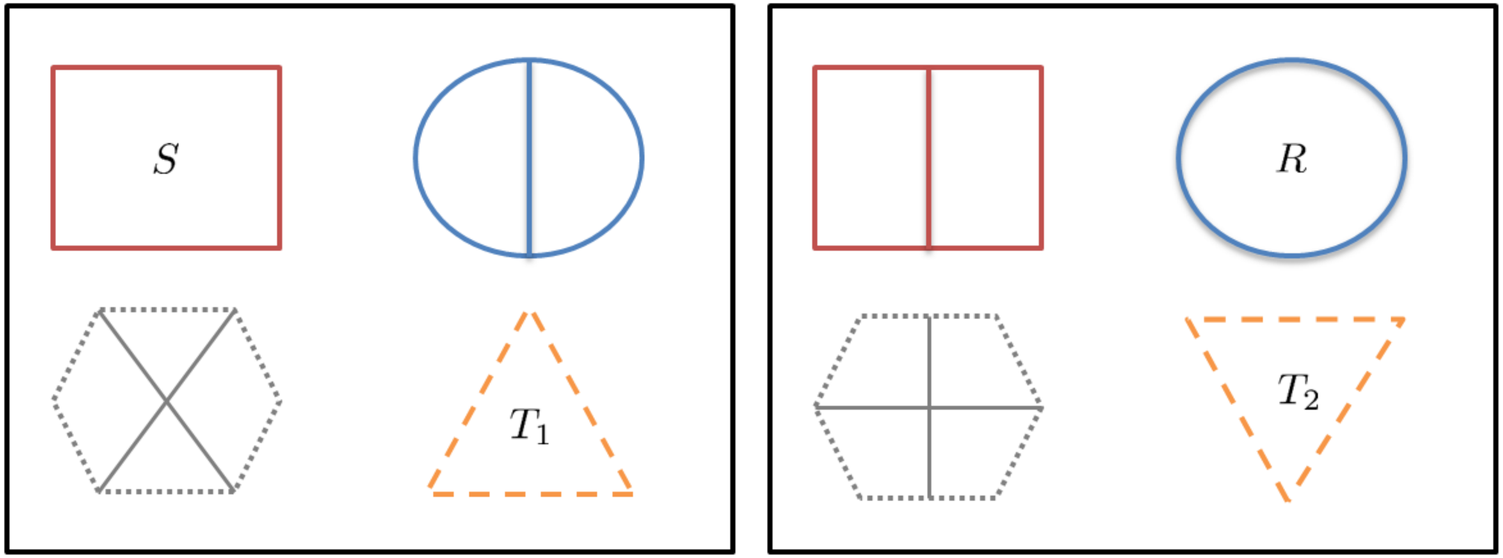
- Uncertain partition boundaries $\Longrightarrow$ many possible partial partitions
- Multivariate segmentations and superpixel segmentations.
- Composition of hierarchical segmentations
- The dynamic program works for the family of braids, and ensures better infimum for over composition of hierarchies with non-trivial monitors.

No single GT is a refinement of the mean-shift segmentation, but their suprema are!



[Unnikrishnan et al. 2007]

Comparing Segmentations



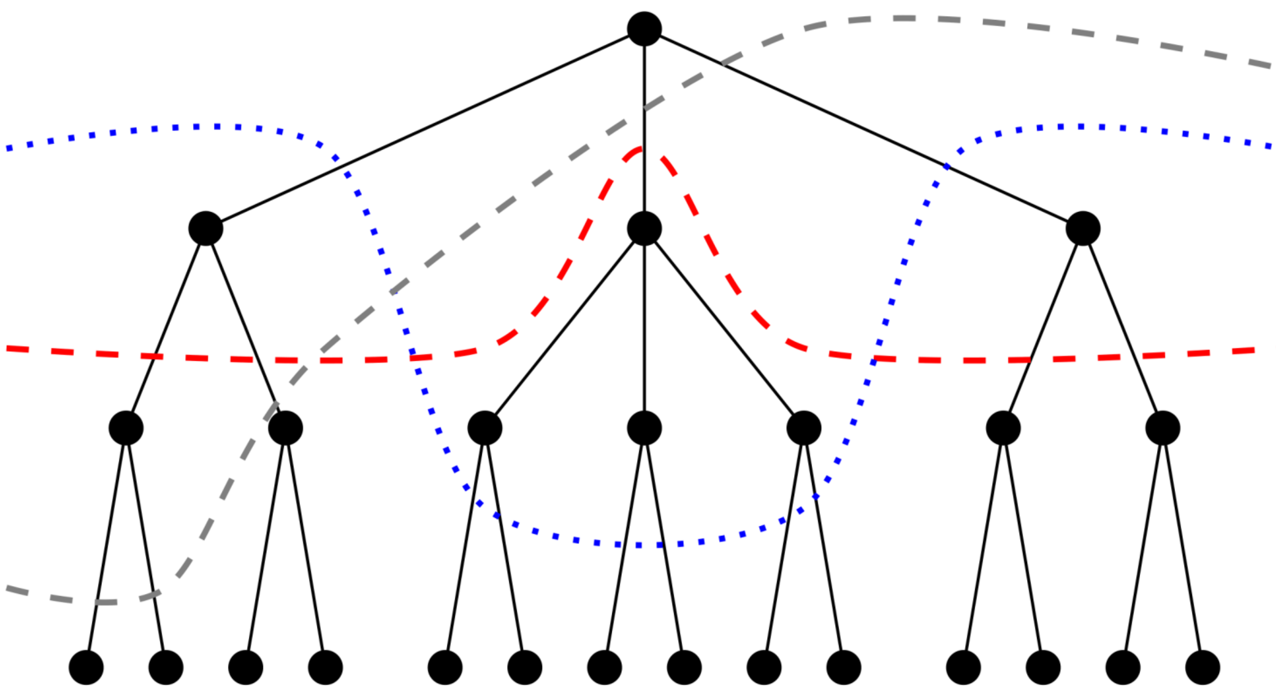
$\pi_1(S) \leq \pi_2(S)$
$\pi_1(R) \leq \pi_2(R)$ } Refinements

$\vee$ $=$ Braid structure

Noise or Overlapp
$T_1 \nsubseteq T_2$ nor $T_2 \nsubseteq T_1$

Braids accomodate variablity in both Expert and Algorithm segmenations

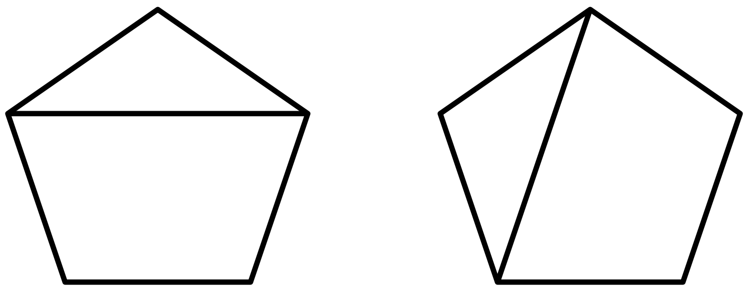
Constraint Partition Sets: Braid Structure

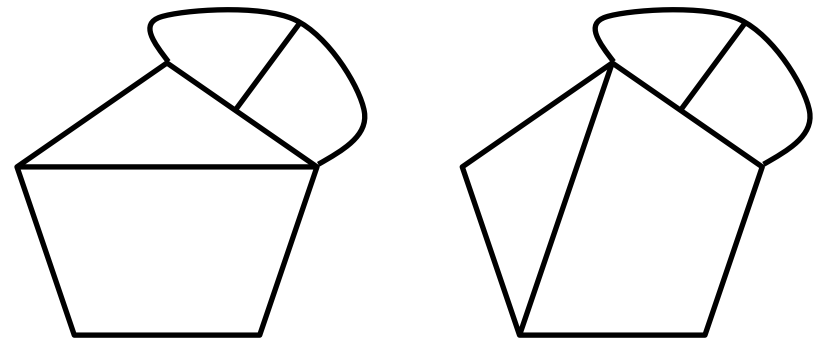


Cuts with $N = 5$

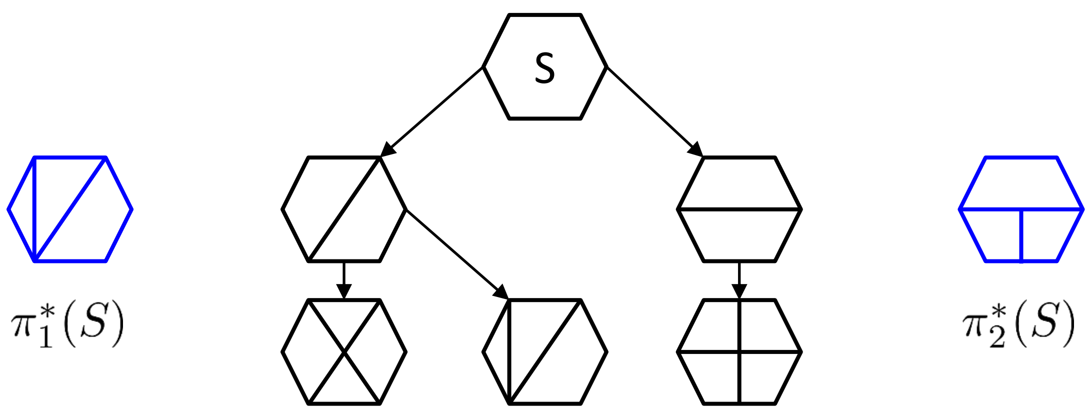
h-increasingness on BOP

$\pi_1(S) \quad \not\leq \quad \pi_2(S)$ $\qquad\qquad$ $\pi_1(S) \sqcup \pi_0 \quad \not\leq \quad \pi_2(S) \sqcup \pi_0$





$$\omega(\pi_1(S)) \leq \omega(\pi_2(S)) \quad \Rightarrow \quad \omega(\pi_1(S) \sqcup \pi_0) \leq \omega(\pi_2(S) \sqcup \pi_0)$$



$\pi_1^*(S)$ $\qquad$ $\pi_2^*(S)$

[PhD Thesis]

Highlights

1. Braids generalize refinement relations on hierarchies

2. Monitor hierarchy might not be part of the Braid itself, but is inherent when calculating supreumum of partitions.

3. Braids provide an algorithm independent partition organization structure.

4. Braids are inherent structures in tree-structured optimization problems

5. Braids still maintain local-global structure for dynamic programs.