UPC / Pompeu Fabra
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LIMG
ESIEE Université Paris-Est

- The hierarchical cuts theory
- Climbing energies and optimal cuts

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The goal

- We have a family of partitions that segment an image.

- How to combine them in order to obtain the best possible segmentation?

- Classically, one associates an energy ω with each partition and one takes the partition with smallest energy (e.g. Mumford-Shah).

  What does this mean really?

Unicity problem

- For example, let us take a small 5×5 picture and an energy whose dynamic range is 1000.

- As there are 4.6×10^{18} different partitions of the 5×5 square, one finds on average :

  4,600,000,000,000,000 partitions by energy !

  i.e. 30 billions times the distance to the moon in kilometres :)

- The methods which work well introduce additional implicit assumptions

How to get out ?

- We keep down the number of possible partitions by restricting them to the cuts of a hierarchy.
- We structure these cuts in a lattice which depends on the energy ω, which ensures a unique minimum.
- We must find a way for reaching easily this minimum.
- When there are several energies, or an energy which depends on a positive parameter, we must find out how to combine them.

...that will be the plan of the talk

Plan

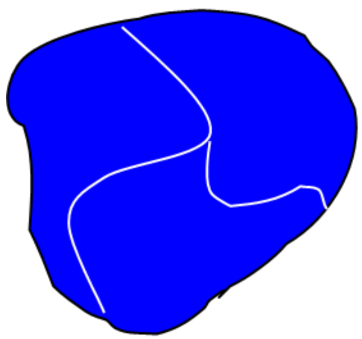
1. Hierarchies
2. Singular energies and lattices
3. Optimal cuts and hierarchical increasingness
4. Compositions of energy by sums and by suprema
5. Climbing families of energies.

Hierarchy, or pyramid, of partitions

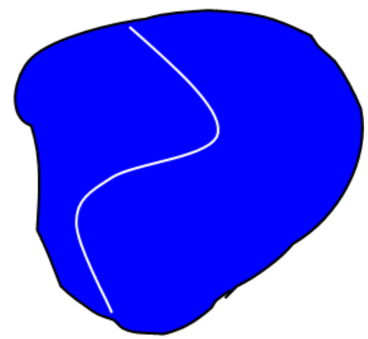
- A hierarchy of partitions is a chain of partitions

$$H = \{\pi_i, 0 \leq i \leq n\} \text{ with } i \leq j \Rightarrow \pi_i \leq \pi_j$$

- The partitions are ordered by refinement



$\leq$



π_1 $\qquad$ π_2

The assumption: π_0 has a finite number of classes, called leaves.

Hierarchy, or pyramid, of partitions

- Associate with hierarchy H the family $\mathcal{S}$ of all classes $S_i(x)$ for all partitions.

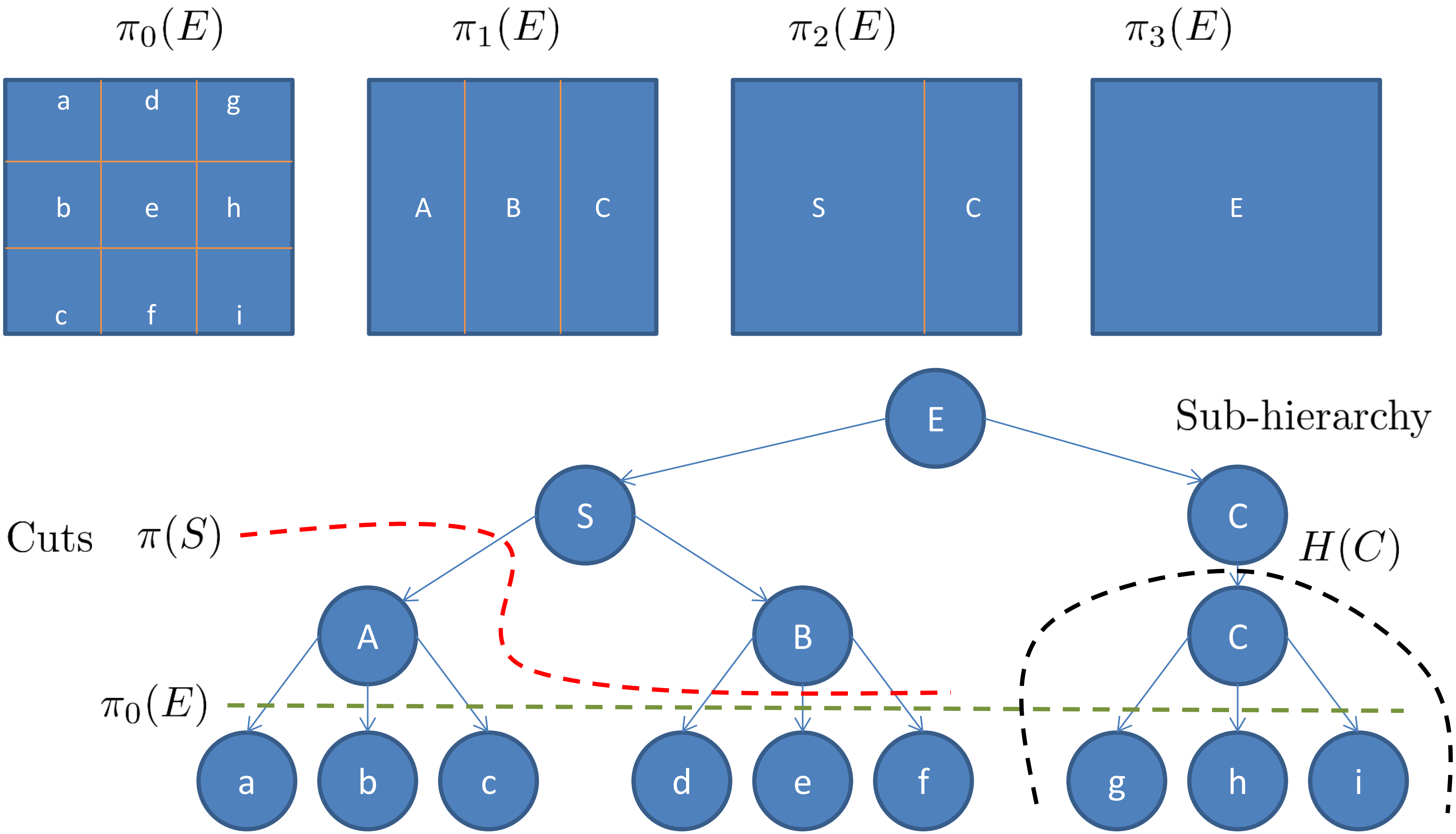
$$\mathcal{S} = \{S_i(x), x \in E, 0 \leq i \leq p\}$$

- Every family $\mathcal{S}$ of indexed sets induces a hierarchy iff for $i \leq j$

$$x, y \in E \Rightarrow S_i(x) \subseteq S_j(y) \quad \text{or} \quad S_i(x) \supseteq S_j(y) \quad \text{or} \quad S_i(x) \cap S_j(y) = \emptyset$$

A relation equivalent to an ultra-metric on the classes of $\mathcal{S}$.

Representation of a hierarchy



Energy and pyramid

The search for an optimal cut rests on three independent entities:

- a pyramid H of partitions of space E

- a function f on E

  (f may have been used, or not, to generate the pyramid),

- an energy ω i.e. a non negative function

  $\omega : \mathcal{D} \to \mathbb{R}^+$

  of the set $\mathcal{D}$ of the partial partitions of E into $\mathbb{R}^+$.

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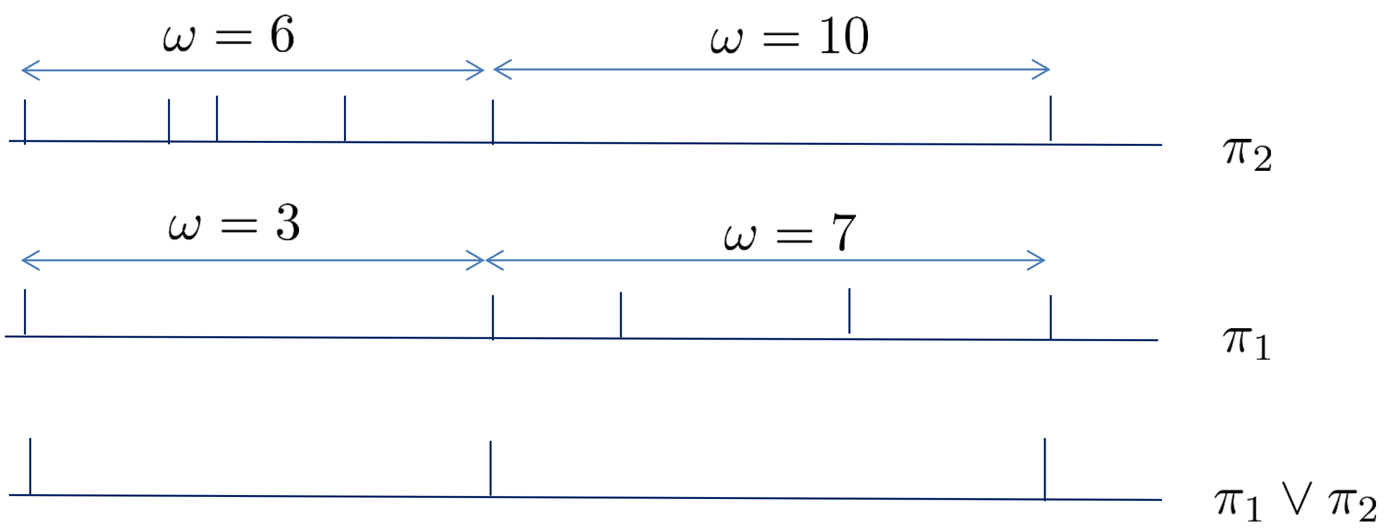
Energetic ordering on cuts

Cut π_1 is said to be less energetic than π_2 when, in each class S of $\pi = \pi_1 \vee \pi_2$ the energy of π_1 in S is smaller than that of π_2 in S.



One writes $\pi_1 \leq_\omega \pi_2$

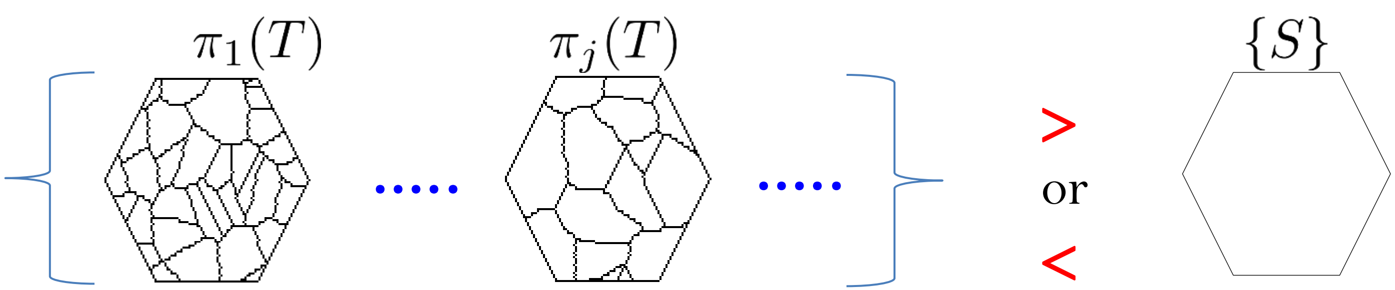
Energetic ordering and singular energy

Is the relation $\pi_1 \leq_\omega \pi_2$ an ordering ?

Proposition: The relation $\pi_1 \leq_\omega \pi_2$ defines an ordering, called energetic, iff the energy ω is singular.

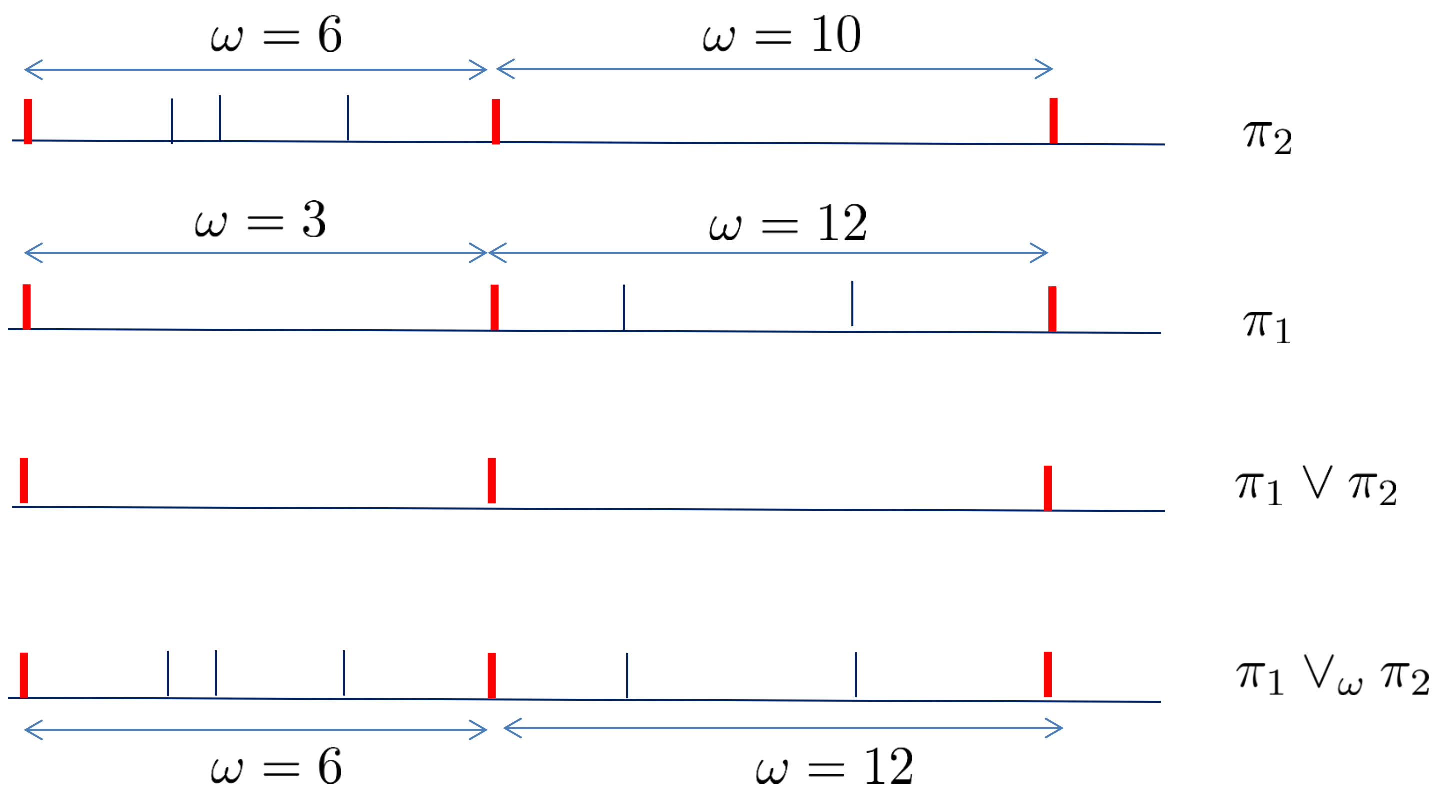
Energy ω is singular when

- either $\omega(S) > \vee\omega(\pi(S))$
- or $\omega(S) < \vee\omega(\pi(S))$



Energetic Lattice

The energetic ordering induces a lattice where, in each class of $\pi_1 \vee \pi_2$ the most energetic partial partition is chosen.



Energetic Lattice

- The energetic lattice ($\leq_\omega, \vee_\omega$) answers the unicity question, since:

  When an energy is singular then one cut only has a minimum energy.

- In this optimal cut, each class S is less energetic than all possible partial partitions of support S.

- Such a minimum is thus stronger than the usual energetic minima since it is both local and global.

- It just remains to find out how to get it :)

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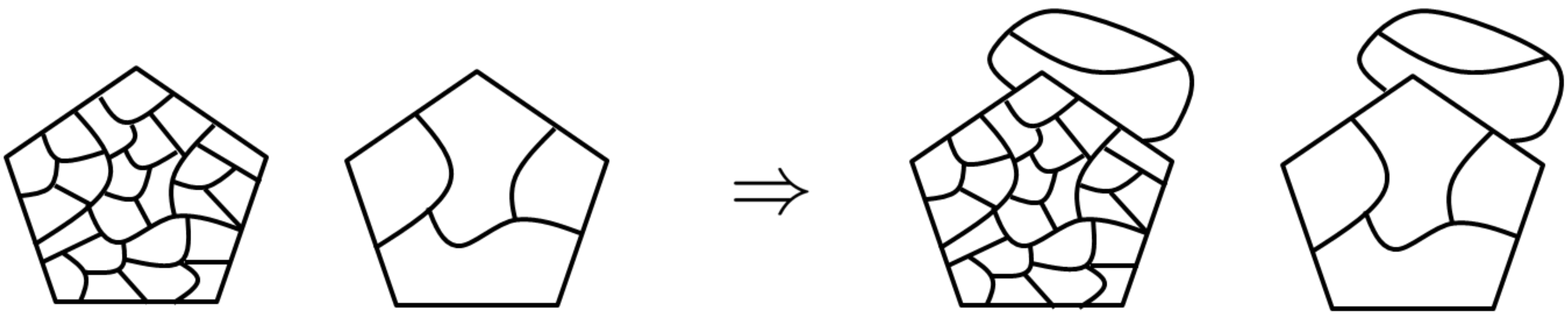
Hierarchical increasingness

- How to reach the cut of minimal energy ?
- Introduce the hierarchical increasingness (h-increasingness) axiom between fathers and sons, as the implication:

$\pi_1(S)$ $\quad$ $\pi_2(S)$ $\quad$ $\pi_1(S) \sqcup \pi_0$ $\quad$ $\pi_2(S) \sqcup \pi_0$



$$\omega(\pi_1(S)) \leq \omega(\pi_1(S)) \qquad \omega(\pi_1(S) \sqcup \pi_0) \leq \omega(\pi_2(S) \sqcup \pi_0)$$

Climbing energies

Energy is said to be climbing when it is both

- Singular (unique optimal cut), and
- h-increasing (tractable access to the optimal cut).

- **Proposition:** When energy ω is climbing then the optimal cut of the sub-hierarchy $H(S)$ is

  either $\pi(T_1) \sqcup \pi(T_2) \sqcup \pi(T_3)$ or S itself

- The optimal cut for the whole space E is then obtained by progressively climbing from the leaves level to the root.

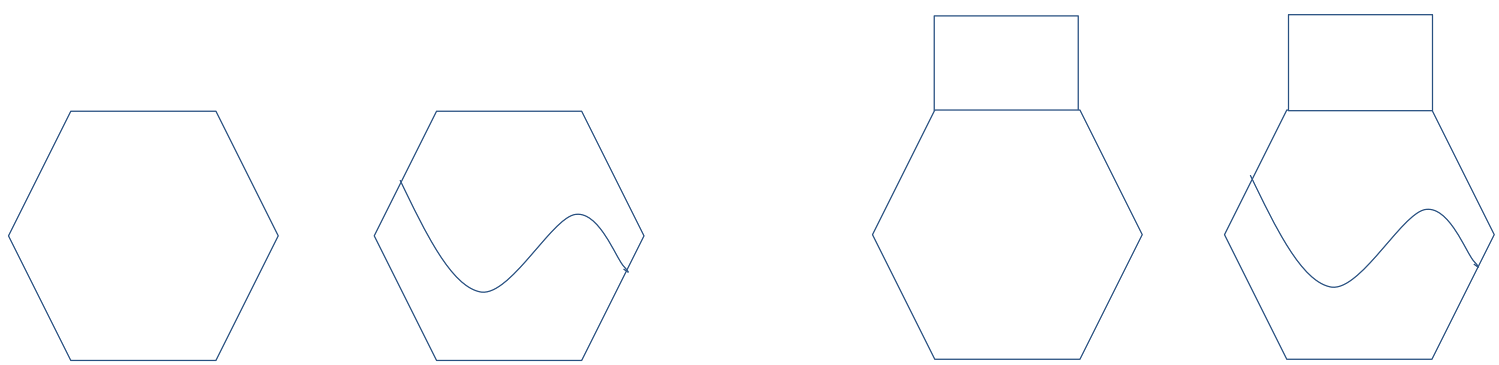
Hierarchical increasingness

- The energies holding on partial partitions are far from being always h-increasing.
- Consider the partial partitions of support S.

  $\omega(\pi(S)) = 1$ when $\pi(S)$ has atmost two components,

  $\omega(\pi(S)) = 0$ when $\pi(S)$ when not.

  The energy ω above is obviously not h-increasing:

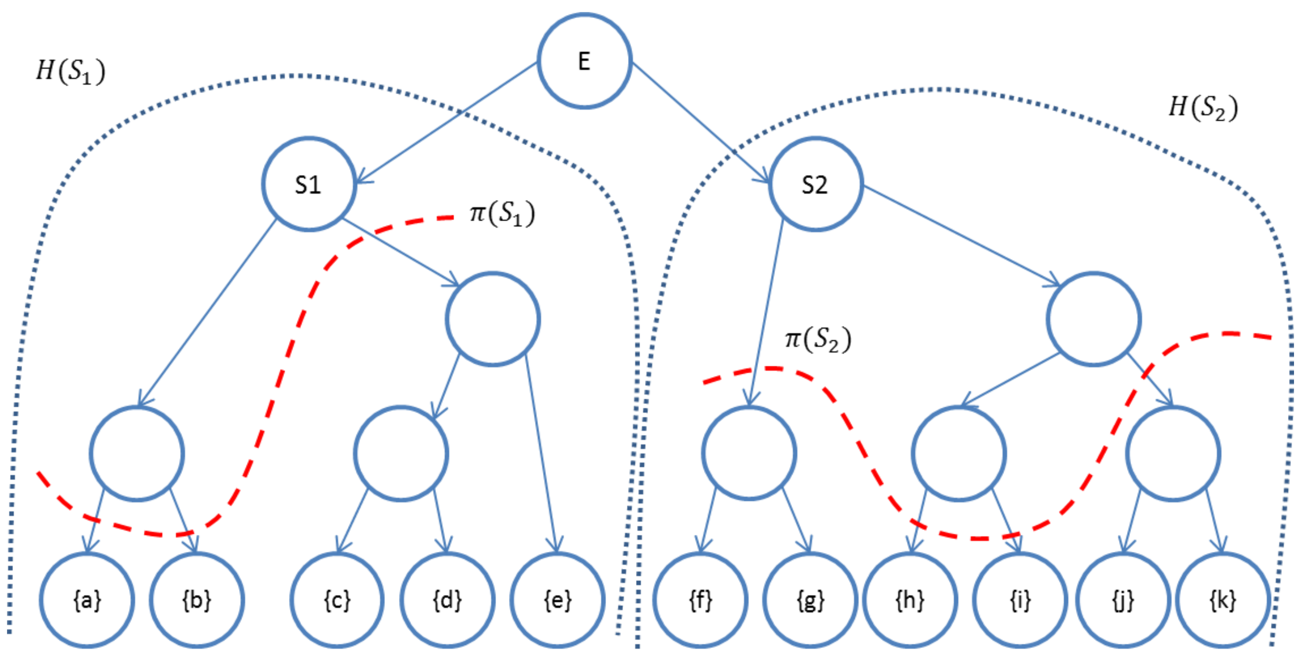


$$\omega = 1 \quad \leq \quad \omega = 1 \qquad\qquad \omega = 1 \quad \not\leq \quad \omega = 0$$

Algorithms

- One scans all nodes of H in one ascending pass according to a lexicographic order of H;

- On determines at each node S a temporary optimal cut of H by comparing the energy of S with those of the (already scanned) sons T_k of S.



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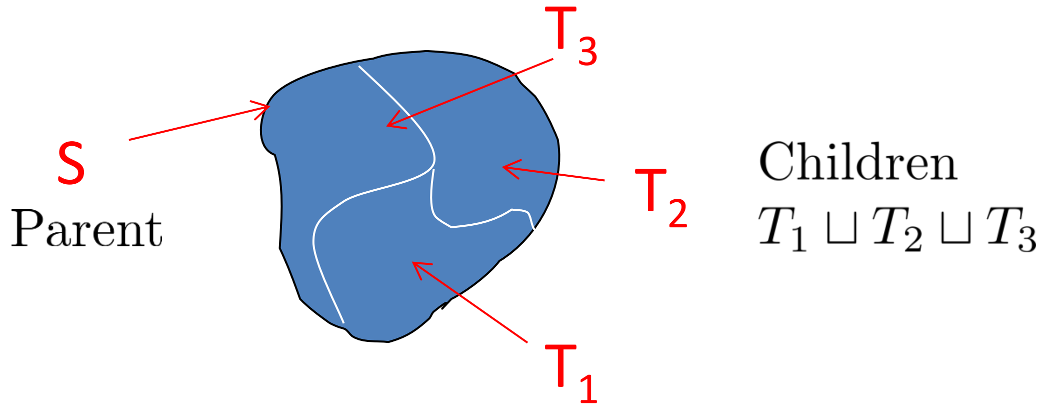
How to construct a climbing energy?

- To get an h-increasing energy, it suffices to start from an arbitrary energy on $\mathcal{S}$



- and to extend it to the partial partitions of support S and of classes T_1, T_2, T_3 by admissible composition rules, e.g.

$$\omega(\pi) = \omega(T_1) + \omega(T_2) + \omega(T_3) \text{ or } \omega(\pi) = \omega(T_1) \vee \omega(T_2) \vee \omega(T_3)$$

How to construct a climbing energy?

- Examples of h-increasing energies:

  Addition: Mumford-Shah: Salembier, Guigues

  Supremum: Soille-Grazzini, Akcay-Aksoy, wavelets.

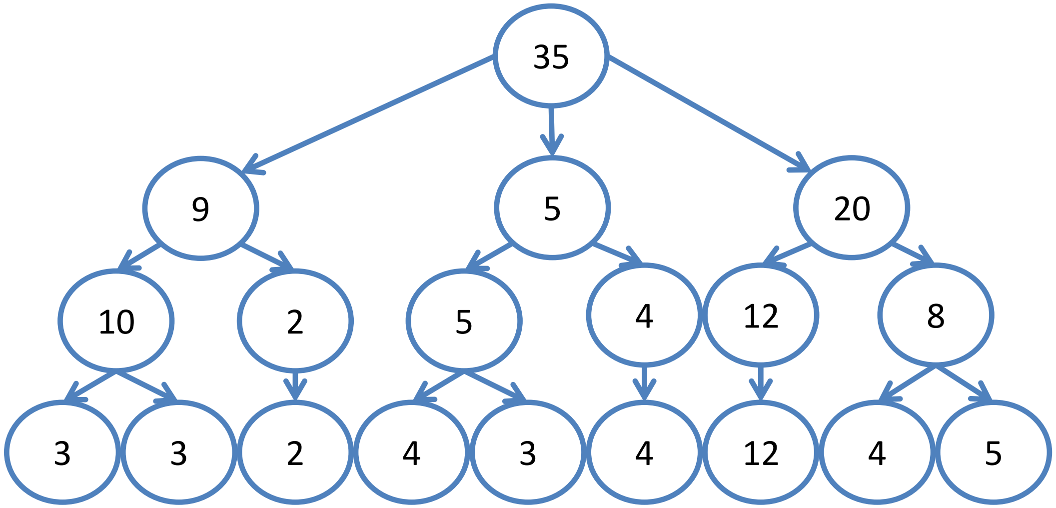
- When ω is h-increasing, and when

  $\omega(T_1 \sqcup T_2 \sqcup T_3) = \omega(S)$

- then we generate a climbing energy by taking either the father or the sons by any external constraint independent of ω

- For example, by taking always the father, or choosing according to the number of sons (e.g. textures).

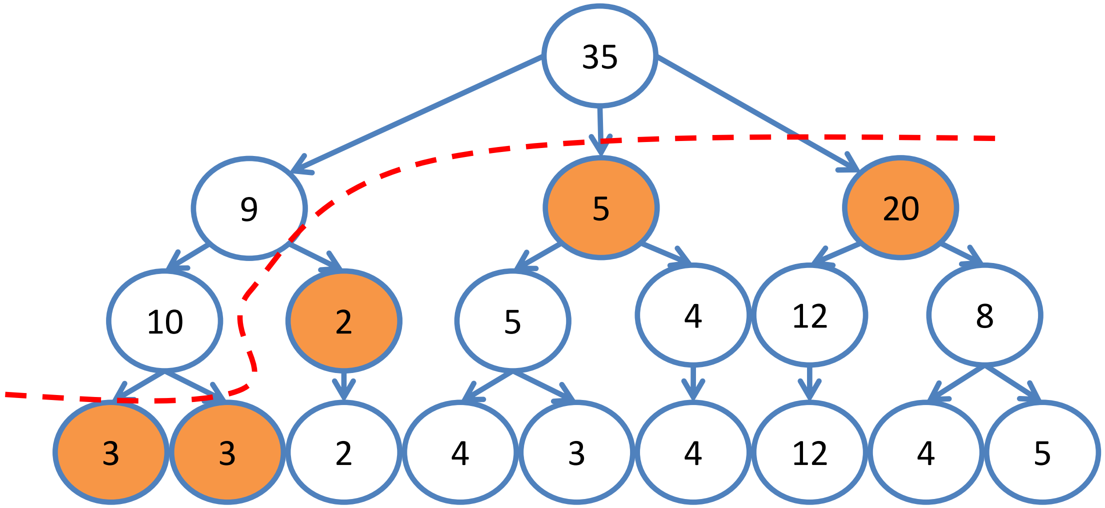
Sum generated energies (Salembier-Guigues)



- The value $\omega(S)$ at node S is compared to the sum $\sum_k \omega(T_k)$ of the energies of the sons:
- if $\omega(S) \leq \sum_k \omega(T_k)$, one keeps the class S,
- if not replace by its sons

The optimal cut is then the union of the remaining classes.

Sum generated energies (Salembier-Guigues)

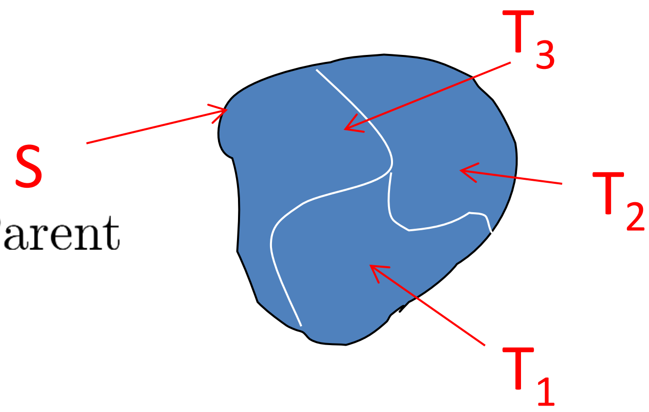


- The value $\omega(S)$ at node S is compared to the sum $\sum_k \omega(T_k)$ of the energies of the sons:
- if $\omega(S) \leq \sum_k \omega(T_k)$, one keeps the class S,
- if not replace by its sons

The optimal cut is then the union of the remaining classes.

An example : Mumford-Shah

Parent



Children
$T_1 \sqcup T_2 \sqcup T_3$

$$\omega(S, \lambda) = \sum_{1 \leq k \leq p} \omega_\varphi(T_k) + \lambda \sum_{1 \leq k \leq p} \omega_\partial(T_k)$$

with $\omega_\varphi(T) = \int_{x \in T} ||f(x) - \mu(T)||^2$ fidelity term,

and $\omega_\partial(T) = |\partial T|$ regularity term

Optimal Cut: Luminance

Initial Image

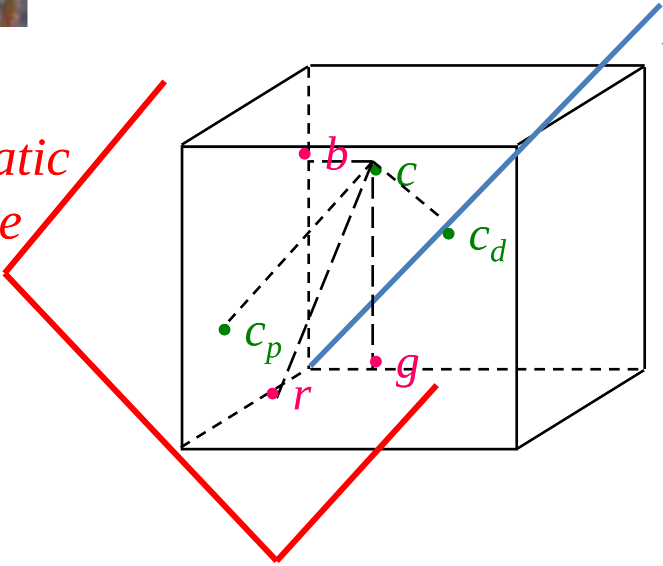
Optimal Cut (Luminance)

$$\omega(S,\lambda) = \sum_{1\leq k\leq p} \omega_\varphi(T_k) + \lambda \sum_{1\leq k\leq p} \omega_\partial(T_k)$$

with luminance $\omega_\varphi(T) = \int_{x\in T} ||l(x) - \mu(T)||^2$ fidelity term.

Luminance-Chrominance

Initial Image



Optimal Cut: Luminance

$$\omega(S,\lambda) = \sum_{1\leq k\leq p} \omega_{\varphi}(T_k) + \lambda \sum_{1\leq k\leq p} \omega_{\partial}(T_k)$$

with luminance (top right)

$\omega_{\varphi}(T) = \int_{x\in T} ||l(x) - \mu(T)||^2$

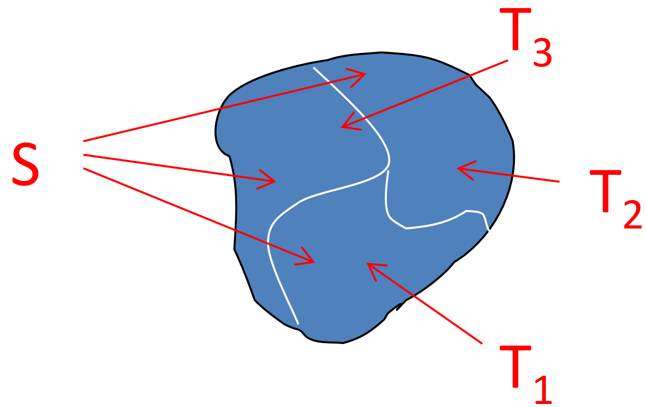
fidelity term.

with chrominance (bottom right)

$\omega_{\varphi}(T) = \sum_i \int_{x\in T} ||c_i(x) - \mu_i(T)||^2$

fidelity term.

Another example: color and texture



$$\omega(S,\lambda) = \sum_{1\le k\le p} \omega_\varphi(T_k) + \lambda \sum_{1\le k\le p} \omega_\partial(T_k) + \mu\omega_\rho(\{T_k\})$$

with chrominance

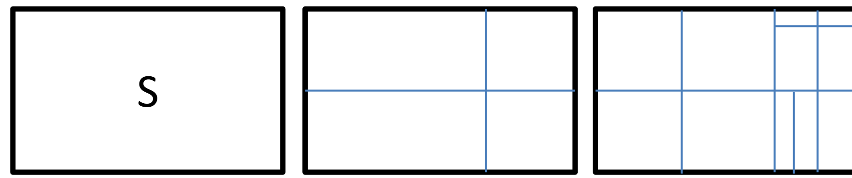
$\omega_\varphi(T) = \sum_i \int_{x\in T} ||c_i(x) - \mu_i(T)||^2$ fidelity term.

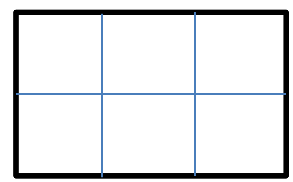
$\omega_\partial(T) = |T|$ Regularization term - contour length

$\omega_\rho(\{T\}) = \sum(\{|T|\} - \mu(\{|T|\}))^2$ Regularization term - texture regularity

Another example: color and texture

Initial Image





Partition with min variation in component sizes

Another example: color and texture

Initial Image

$$\omega(S,\lambda) = \sum_{1\leq k\leq p} \omega_{\varphi}(T_k) + \lambda \sum_{1\leq k\leq p} \omega_{\partial}(T_k) + \mu\omega_{\rho}(\{T_k\})$$

Right: optimal cuts:
top, very uniform textures (high μ)

Another example: color and texture

Initial Image

$$\omega(S,\lambda) = \sum_{1\leq k\leq p} \omega_{\varphi}(T_k) + \lambda \sum_{1\leq k\leq p} \omega_{\partial}(T_k) + \mu\omega_{\rho}(\{T_k\})$$

Right: optimal cuts:
- top, very uniform textures (high μ)
- bottom (weaker μ)

Composition of additive energies

Let $\{\omega_i, i \in I\}$ be a family of additive and singular energies, and $\{\lambda_i, i \in I\}$ a family of positive weights.

Then the weighted sum $\omega = \sum \lambda_i \omega_i$ turns out to be climbing.

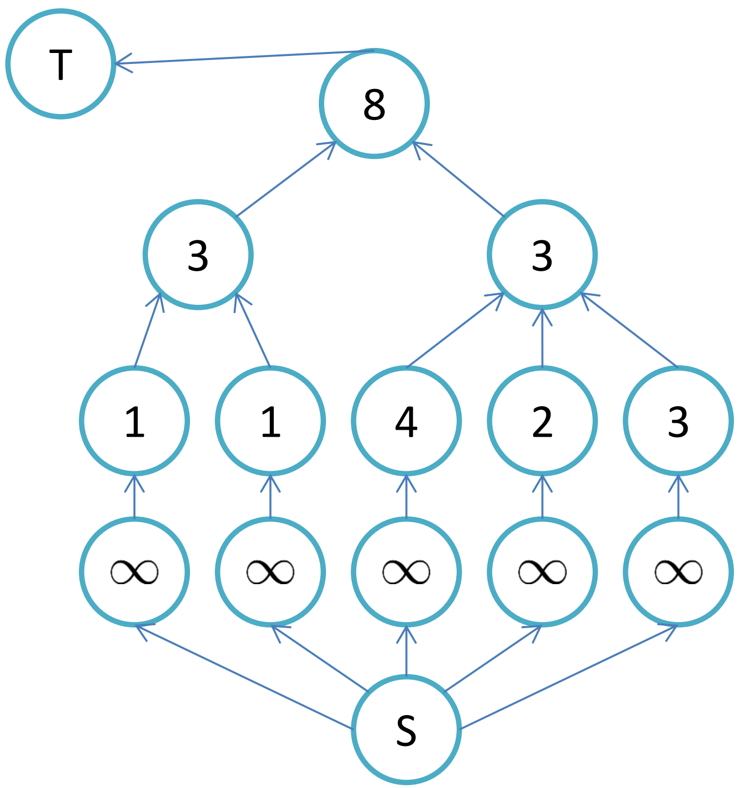
This property allows us to add, or to change, terms in the energies of Mumford-Shah type.

For example, for a color image, if the pyramid is obtained by watersheds of the luminance, use the chrominance for fidelity term.

Additive energies and graph-cuts

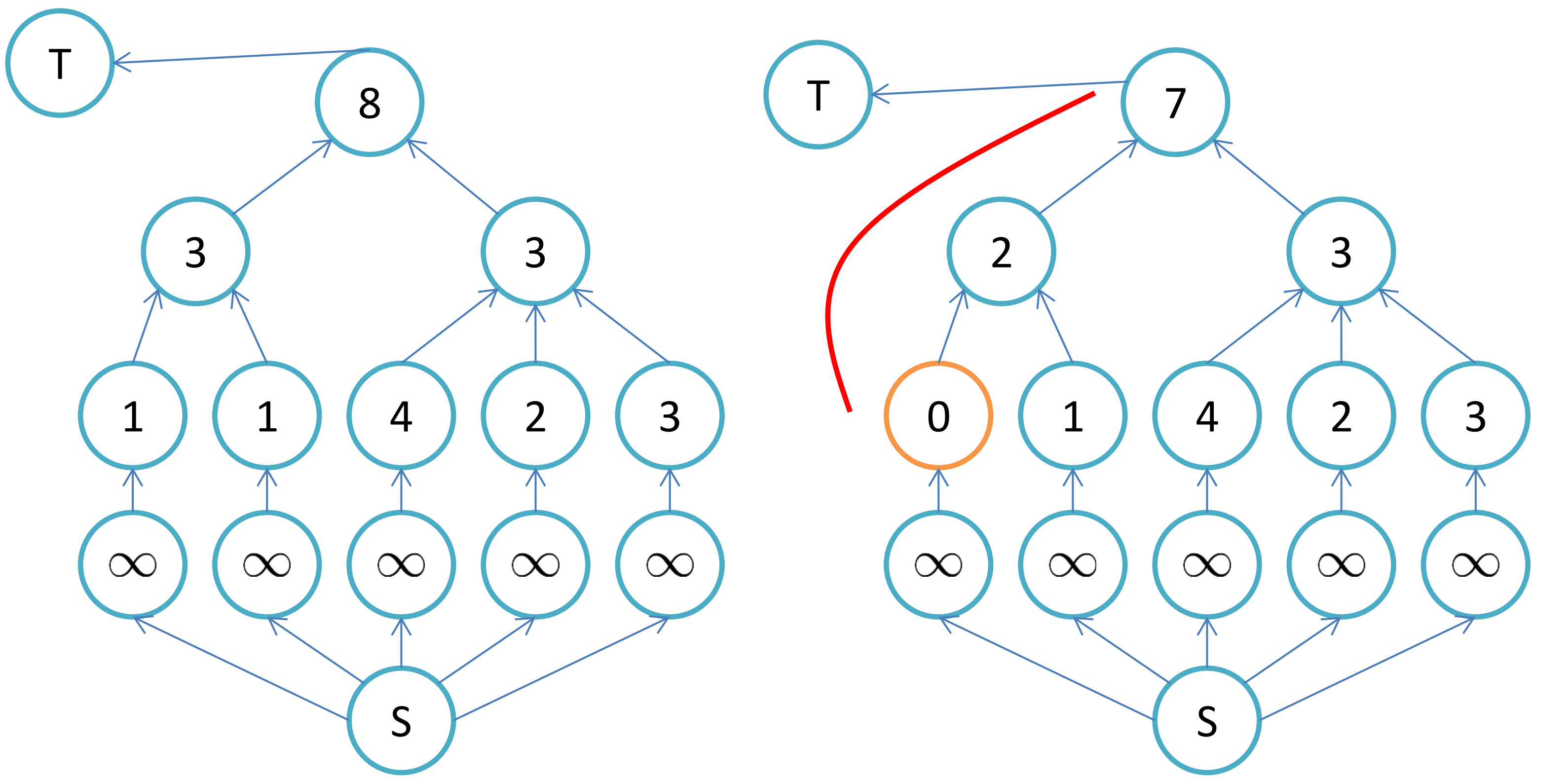
- The definition of a flow through G requires the data of
  - a source: the leaves, with infinite weight,
  - a sink: the root,
  - and a flow capacity at each node.
- The flows of two separated paths are
  - independent,
  - and upper-bounded by the lowest capacity along the path.
- When two lines meet at a (father) node, the capacities of the sons are added and compared to that of the father. On keeps the largest.

Min-cut versus optimal cut



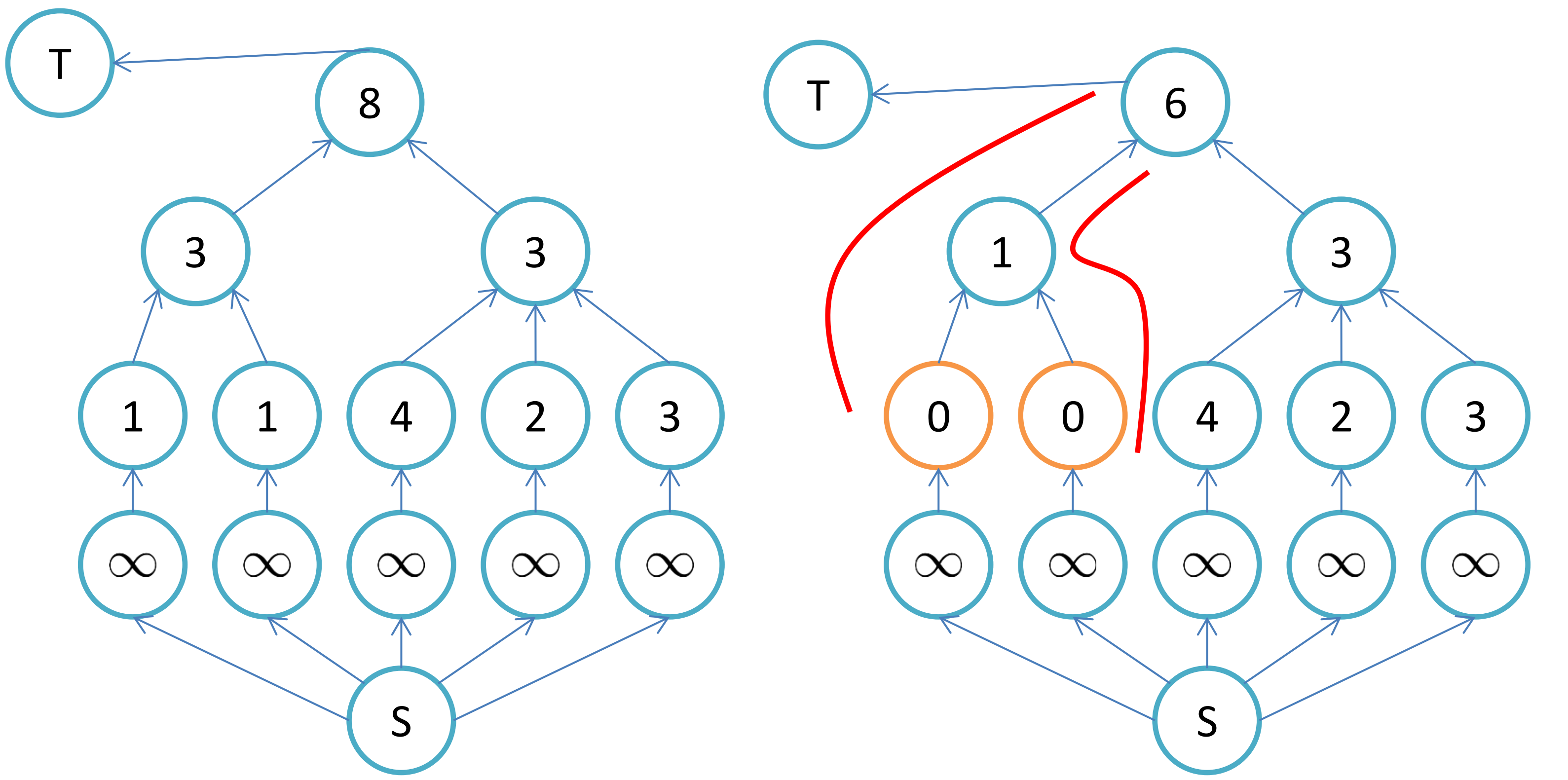
Initial hierarchy

Min-cut versus optimal cut



The minimum value on each path is subtracted from each node in the path, up till the point where we obtain a cut that separates S and T.

Min-cut versus optimal cut



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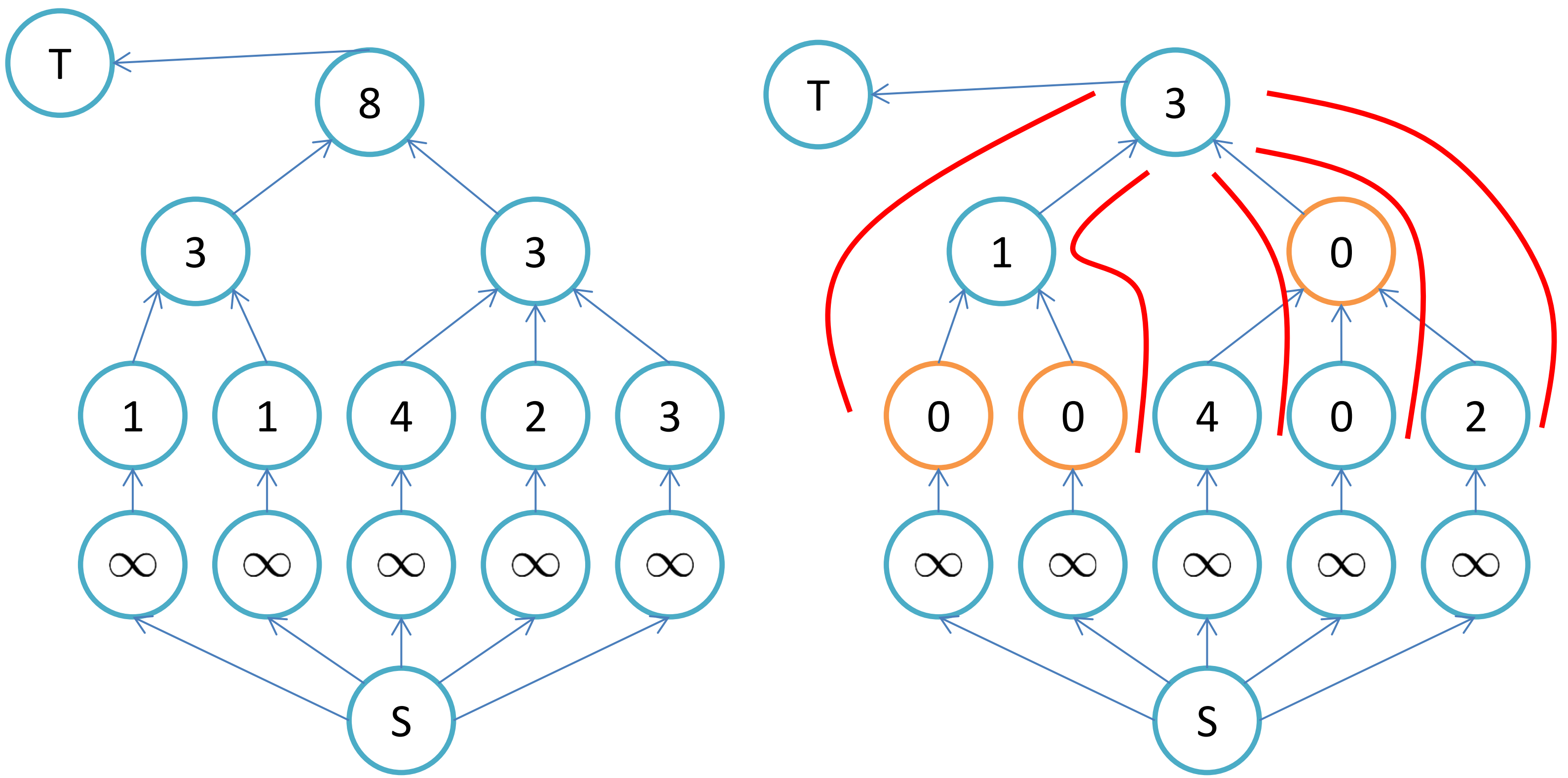
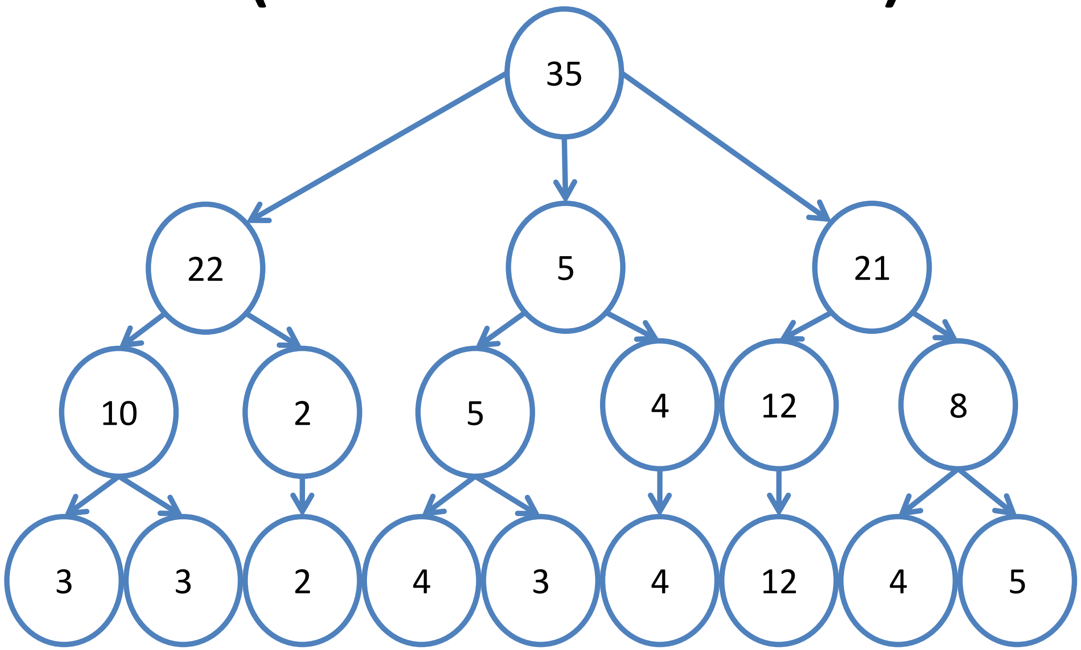
Min-cut versus optimal cut



The minimum value on each path is subtracted from each node in the path, up till the point where we obtain a cut that separates S and T.

The set of saturated nodes(min-cut) is exactly the optimal cut.

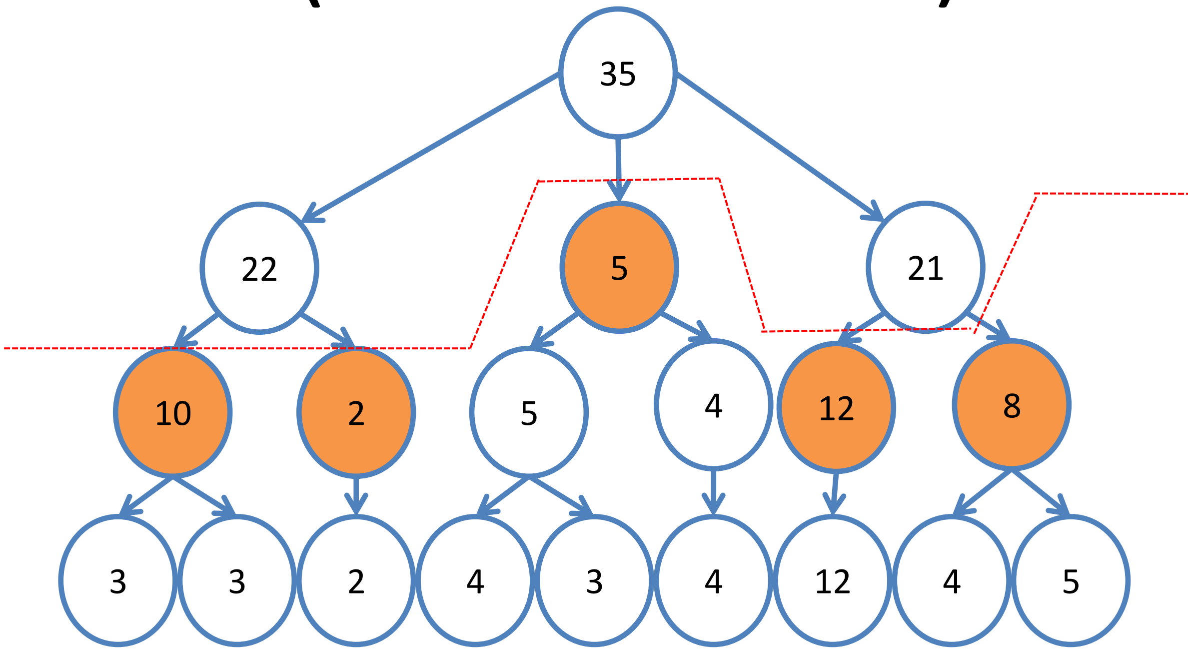
Suprema generated energies (Soille-Grazzini)



- The values of $\omega(S)$ are supposed to increase as going up in the hierarchy. The value at node S is $maxf(S) - minf(S)$.
- Node S is kept when $\omega(S) \leq k$. (here $k = 20$)

The optimal cut is the union of the largest remaining nodes.

Suprema generated energies (Soille-Grazzini)



- The values of $\omega(S)$ are supposed to increase as going up in the hierarchy. The value at node S is $max f(S) - min f(S)$.
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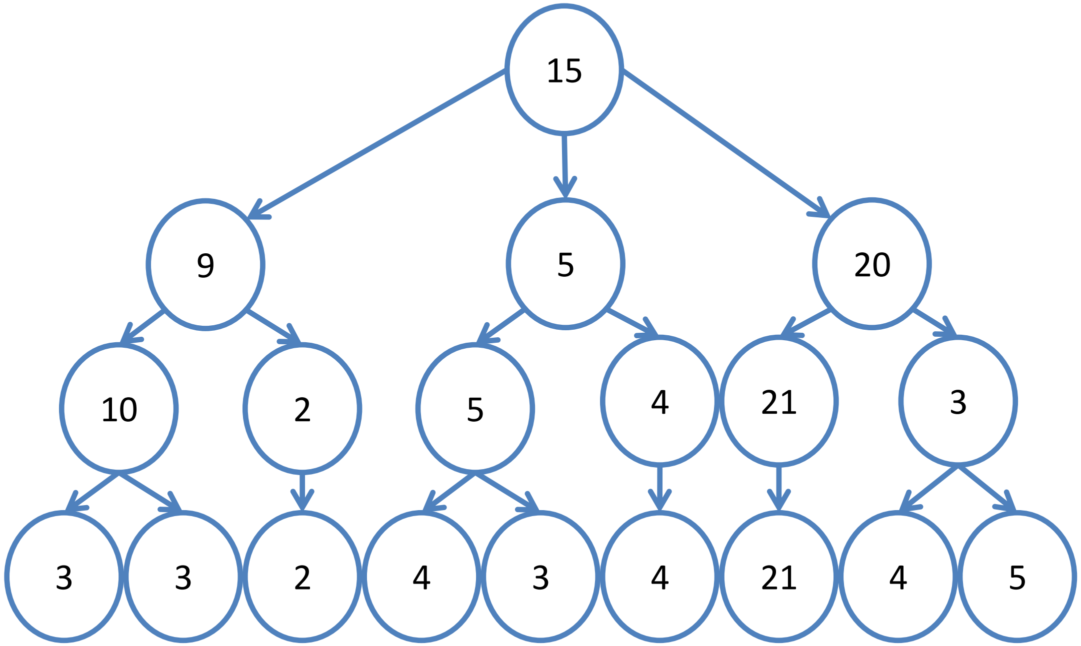
V-generated energies (Akçay-Aksoy)

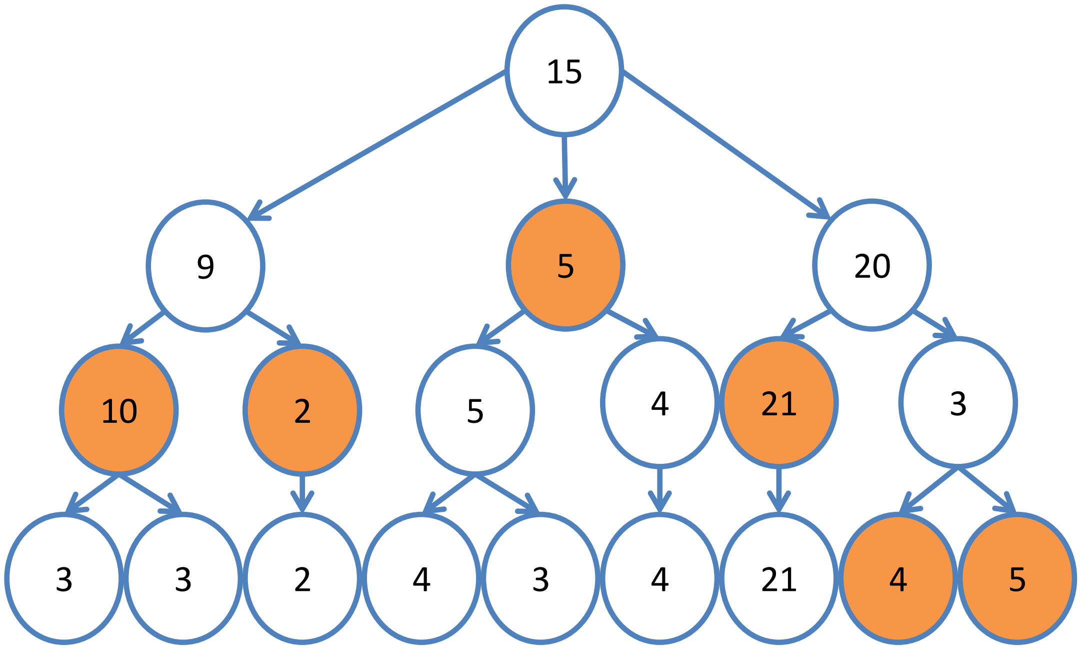


$\omega(S) \leq \omega(S^*)$ when $S \subseteq S^*$ and $f(S) \leq f(S^*)$

The optimal cut at point x made by the set of all nodes more energetic than their descendants,

or, when none, by the leave containing x.

V-generated energies (Akçay-Aksoy)



In this case the optimum is a result of maximization.

$\omega(S) \leq \omega(S^*)$ when $S \subseteq S^*$ and $f(S) \leq f(S^*)$

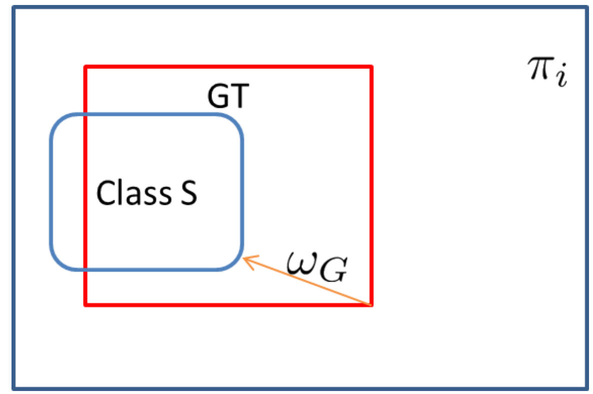
The optimal cut at point x made by the set of all nodes more energetic than their descendants,

or, when none, by the leave containing x.

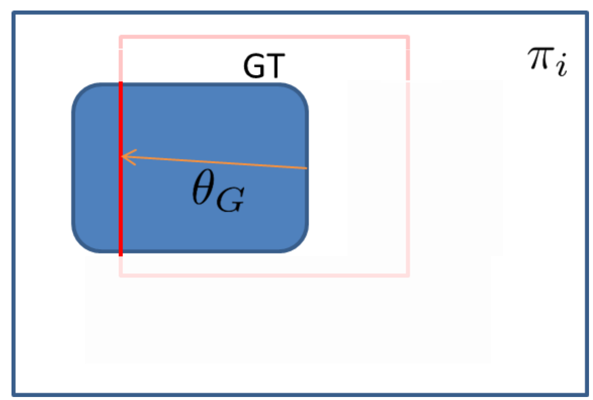
Infima generated energies
Ground truth Evaluation

In the next session we have an example that performs composition by suprema and infima applied to the problem of evaluation of hierarchies by ground truth.

- **Local measures**: Each class S in H is assigned two radii: ω_G and θ_G,
- Given a hierarchy H and ground truth partition G find the partition in H closest to G.
  - Closest from $H \rightarrow G$
  - Closest from $G \rightarrow H$



Min radius of dilation of ground truth contour that covers the contour of S.



Minimum radius of dilation of the contour of S to cover GT within S.

Composition of V-generated energies

- The weighted supremum $\omega = \vee\lambda_i\omega_i$ of a family $\{\omega_i, i \in I\}$, $\{\lambda_i, i \in I\}$ is h-increasing (but not the infimum).

- Note that the supremum can express an intersection of criteria

- For example, if in S

  - $\omega_1(S) = 0$ if the luminance range $< k_1$, and $\omega_1(S) = 1$ if not,
  - $\omega_2(S) = 0$ if the saturation range $< k_2$, and $\omega_2(S) = 1$ if not,
  - $\omega_3(S) = 0$ if the area of S is $\geq k_3$, and $\omega_3(S) = 1$ if not,

  then the energy $\vee\omega_i(S) = 0$ when S is not too small and rather constant in luminance and saturation.

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Climbing families of energies

- The energy often depends on a positive parameter, i.e. $\{\omega^\lambda, \lambda > 0\}$. Is it then possible to order the optimal cuts according to λ?

- The family $\{\omega^\lambda, \lambda > 0\}$ is said to be climbing when:

  each ω^λ is climbing (i.e. singular and h-increasing) for any partial partition π of support S we have

  $\lambda \leq \mu$ and $\omega^\lambda(S) \leq \omega^\lambda(\pi) \Rightarrow \omega^\mu(S) \leq \omega^\mu(\pi)$

Proposition: When the family $\{\omega^\lambda, \lambda > 0\}$ of energies is climbing, then the optimal cuts increase with λ (for the refinement ordering)

Another Example

Initial image

Chrominance as fidelity term

$\lambda = 0$

N.B. no regularity term

Another Example

Initial image

Chrominance as fidelity term

$\lambda = 400$

Another Example

Initial image

Chrominance as fidelity term

$\lambda = 10,000$

Note: the textures have been filtered out.

Conclusions

- We replaced the numerical approach

energy $\Rightarrow$ ***optimal partition***

by the lattice one

energy + singularity $\Rightarrow$ ***Lattice of the cuts*** $\Rightarrow$ ***optimal partition***

which adds a local meaning to the global energy ω, (similar to the uniform convergence versus the simple one).

Conclusions

- We replaced the variational approach by the axiomatics

Singular and h-increasing energy = climbing energy

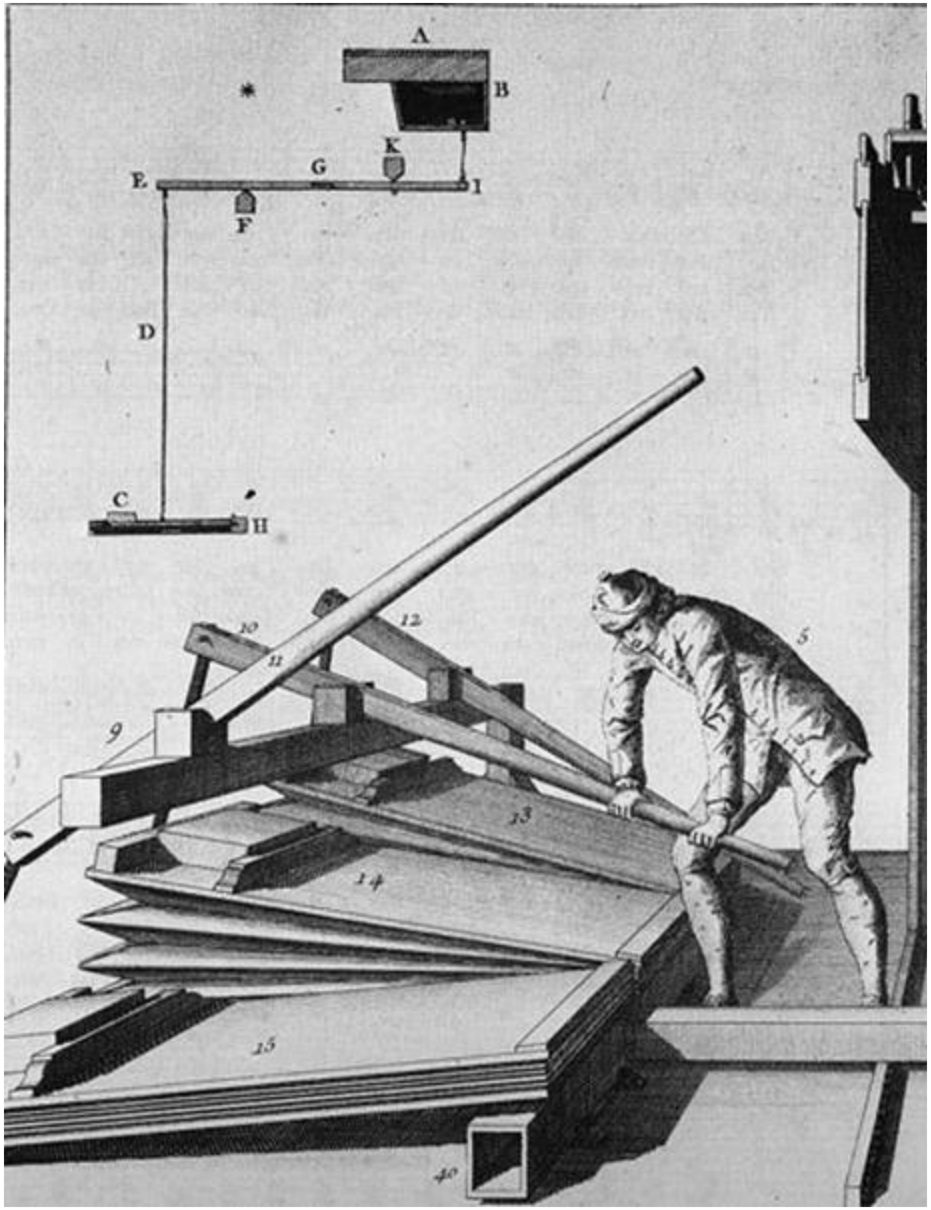
which allows the fast computation

climbing energy ⇨ ***optimal cut in one pass***

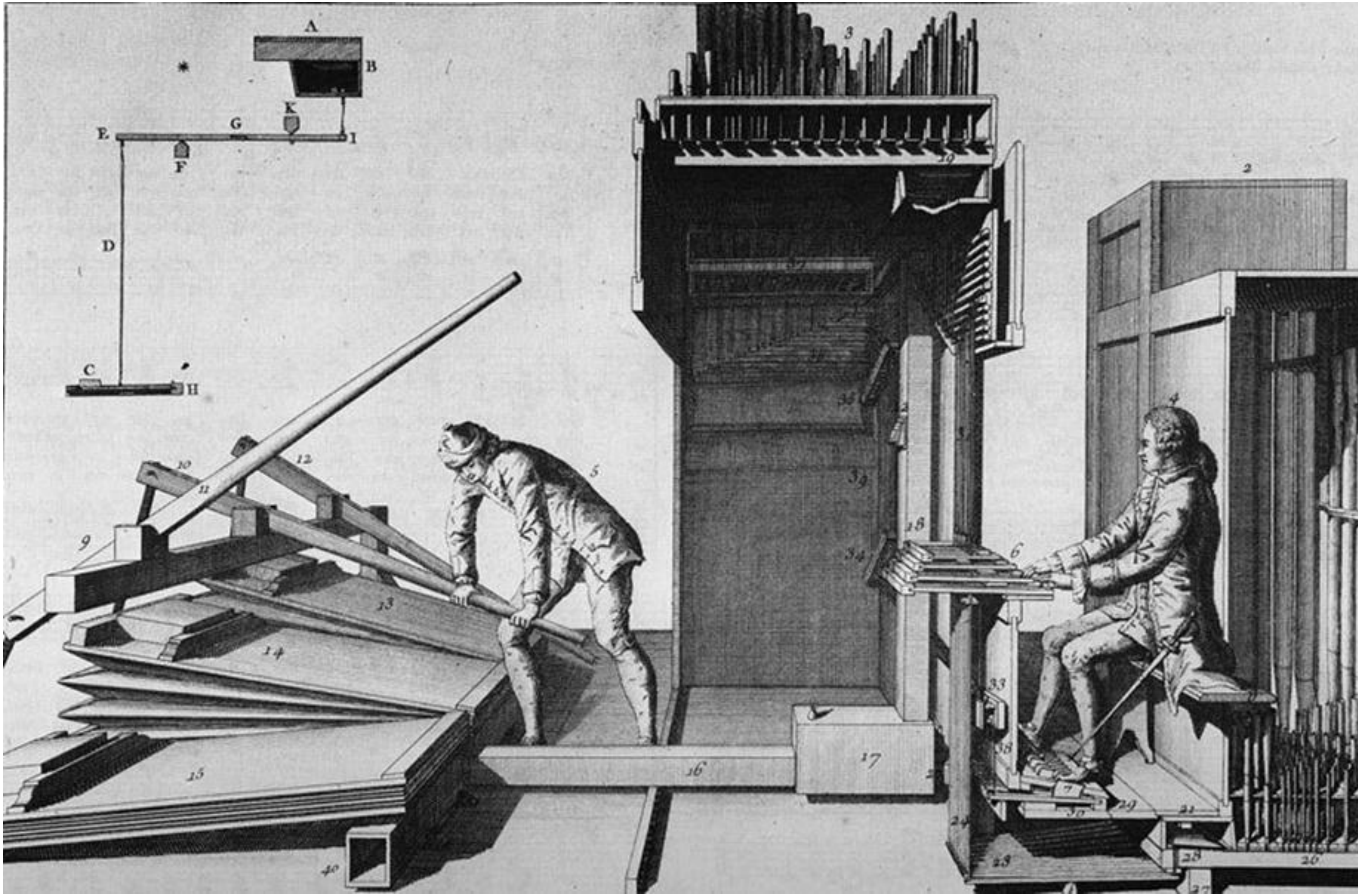
- We introduced the climbing families of energies

Which results in

Climbing families of energies ⇨ ***Hierarchies of optimal partitions***



…. A study by ***my student***



…. A study by ***my student***, and ***me***.

Thanks

For your

attention !

UPC / Pompeu Fabra
Barcelona, June 13 2013

LIMG
ESIEE Université Paris-Est

II- Ground truth energies and Saliency Transform

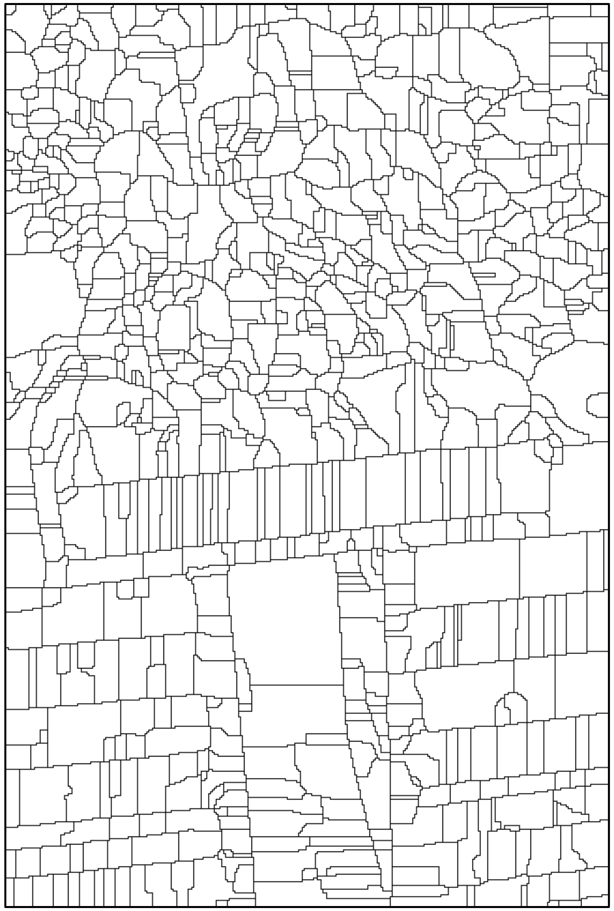
B. Ravi Kiran
Jean Serra

Problem context

1. Developing the theory of optimal cuts. (Pattern Recognition Letters Journal 2013)
2. Ground truth energies (ISMM 2013)
3. Saliency transforms (SSVM 2013)

A Problem: Inputs

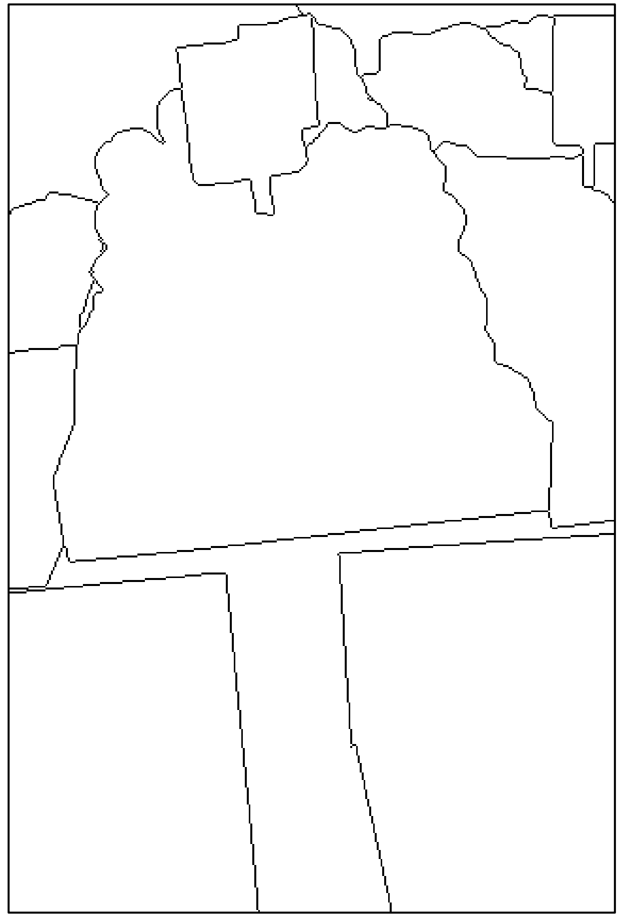
Input Image 25098 Berkeley Database



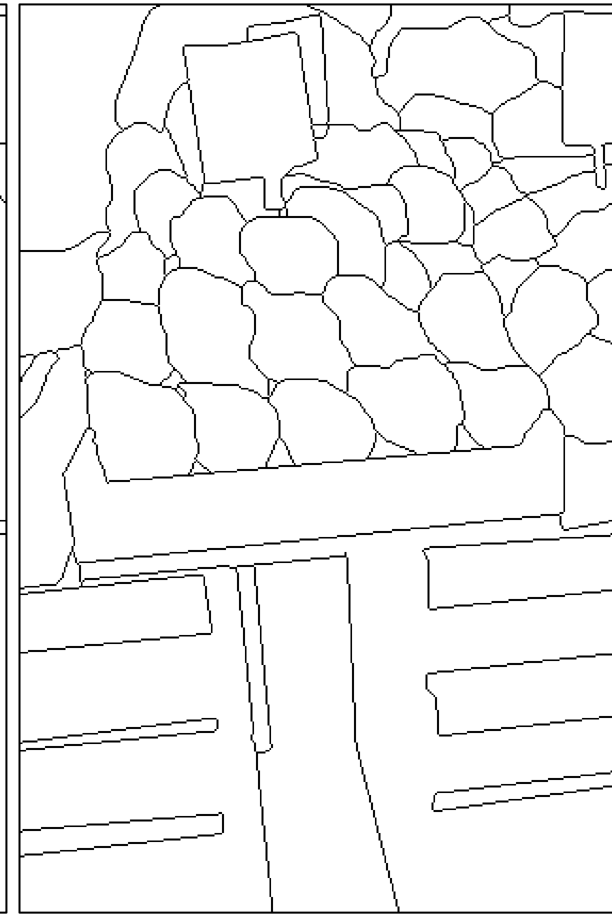
Partitions in input hierarchy of segmentations H

Ground truth: Evaluation of Hierarchies

Input Image



G_2



G_7

Hand drawn ground truth by multiple users or experts for each image. No inclusion ordering assumed in the ground truths.

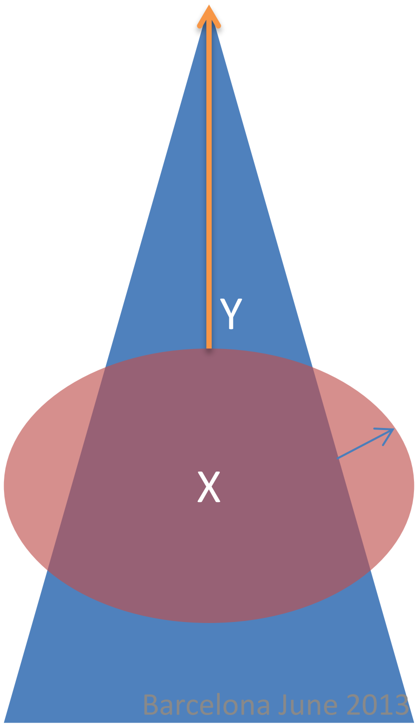
Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
    1. Closest from H -> G
    2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Hausdorff distance and associated problems

$$\mathrm{d}_H(X,Y) = \max\left\{\sup_{x\in X}\inf_{y\in Y}\mathrm{d}(x,y), \sup_{y\in Y}\inf_{x\in X}\mathrm{d}(x,y)\right\}$$

i.e. smallest disc dilation of X that contains Y and of X to contain Y



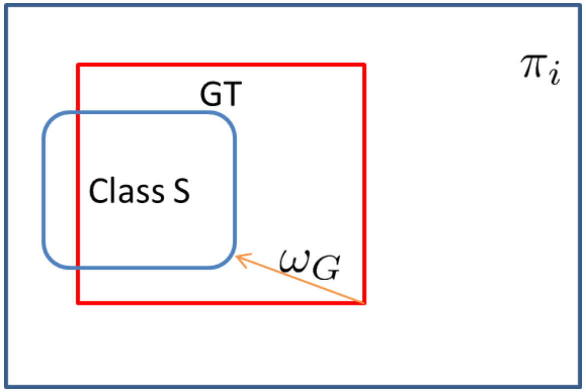
- Global Measure
- Large variations when object are asymmetric w.r.t each other

Local Hausdorff distances

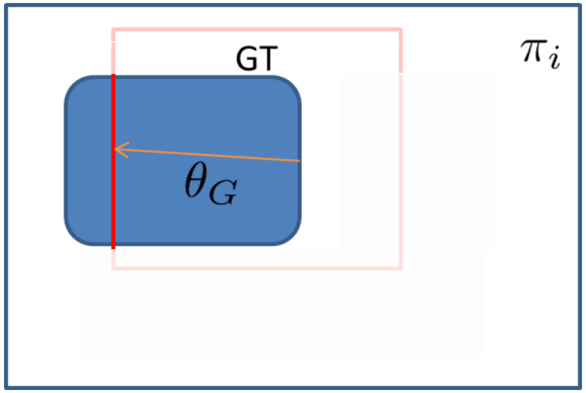
- *Local measures*: Each class S in H is assigned 2 radii: ω_G , θ_G

- Both are h-increasing energies

- Local optimization to obtain a globally optimal solution



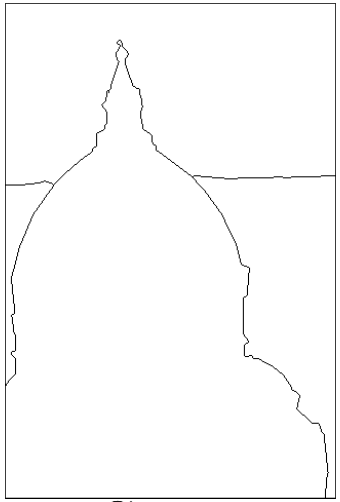
minimum radius of dilation of ground truth contour that covers the contour of S.



minimum radius of dilation of the contour of S to cover GT within S.

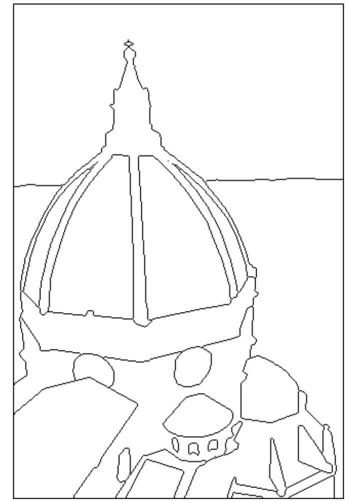
Example

Image

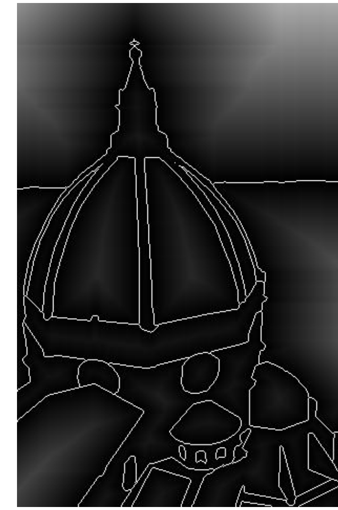


GT2

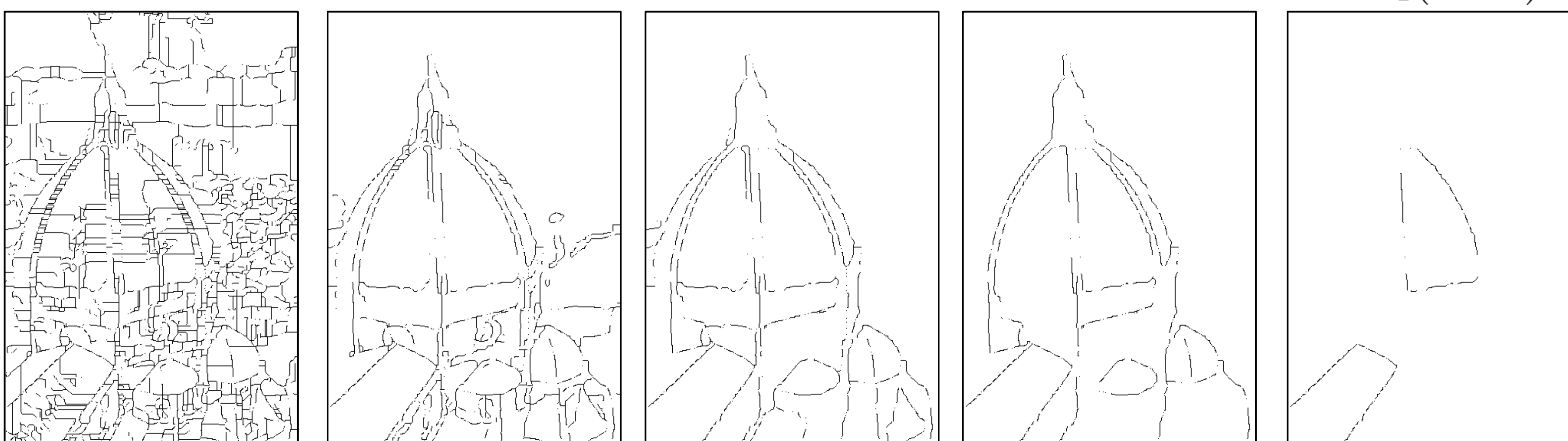
$D_4(GT2)$



GT5

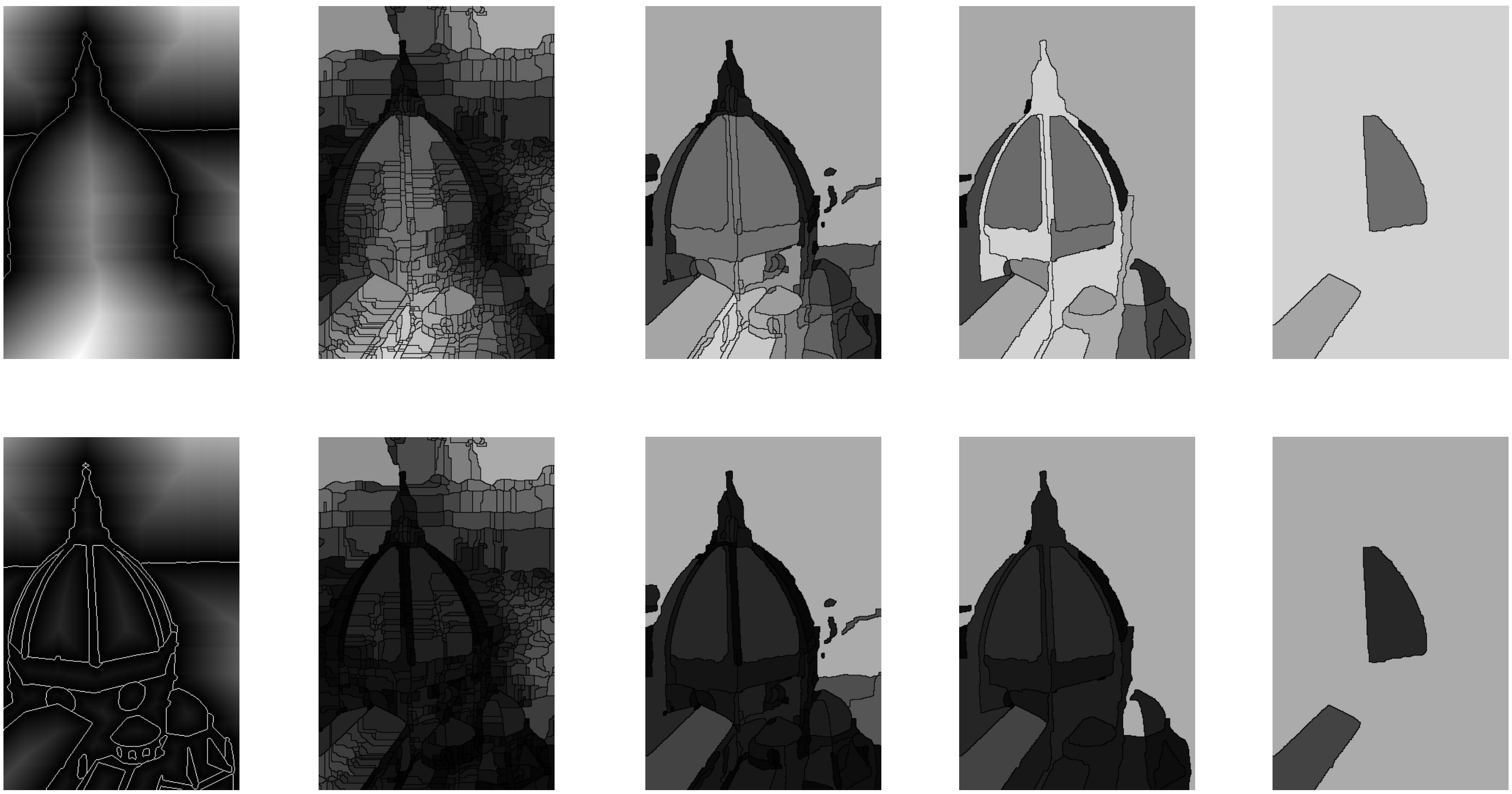


$D_4(GT5)$

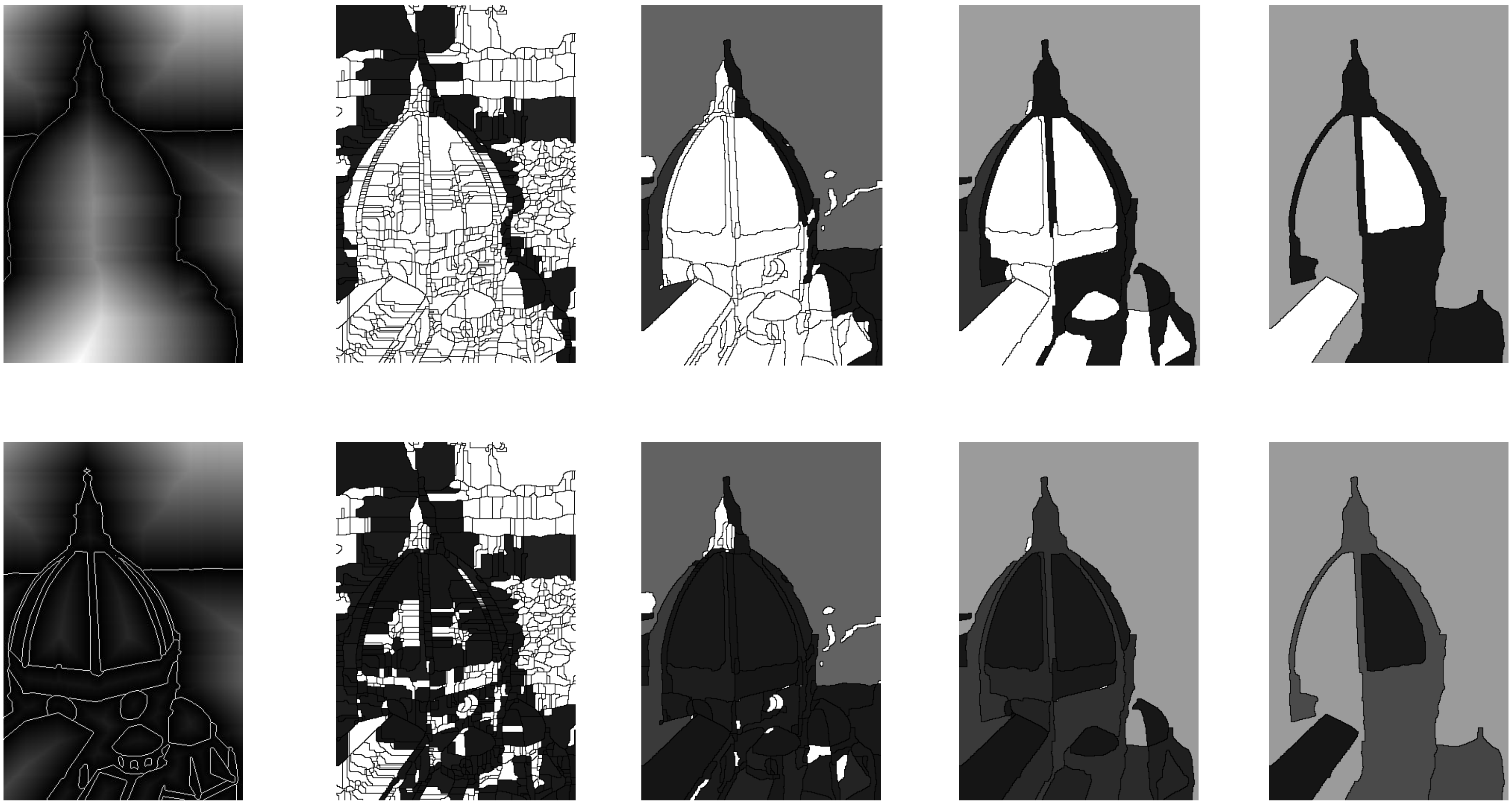


Partitions from thresholding UCM

ω_{G} Energy at various levels of H



θ_{G} Energy at various levels of H

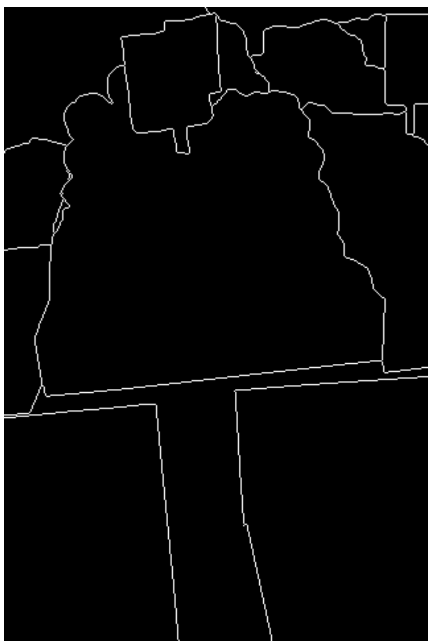


Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
    1. Closest from H -> G
    2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Optimal Cuts

Initial Image



GT2

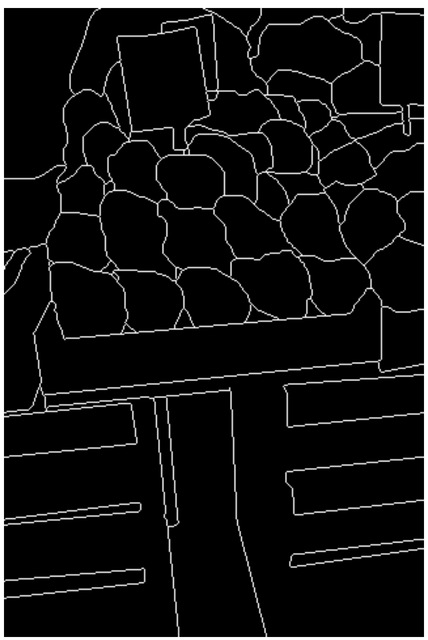
ω_{GT2}

θ_{GT2}

$\omega_{\mathrm{GT2}} + \theta_{\mathrm{GT2}}$

Optimal Cuts

Initial Image



GT7

ω_{GT7}

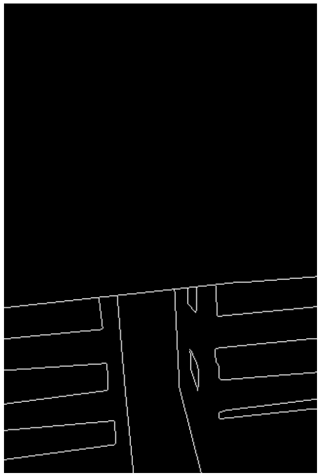
θ_{GT7}

$\omega_{\mathrm{GT7}} + \theta_{\mathrm{GT7}}$

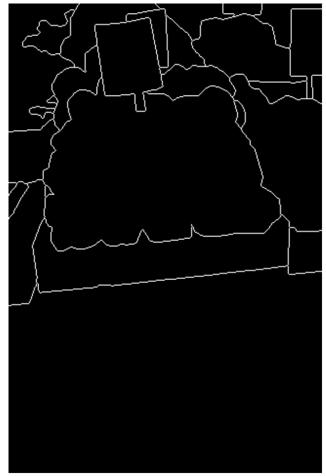
Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
    1. Closest from H -> G
    2. Closest from G -> H
2. Compare any hierarchy H with multiple ground truth partitions of the same image
3. Compare any two hierarchies H1, H2, with respect to a common ground truth partition G

Composition of ground truths:

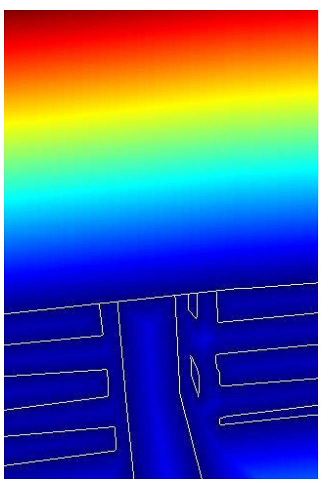


GT5_1

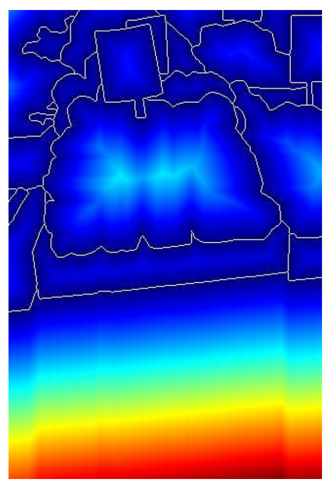


GT5_2

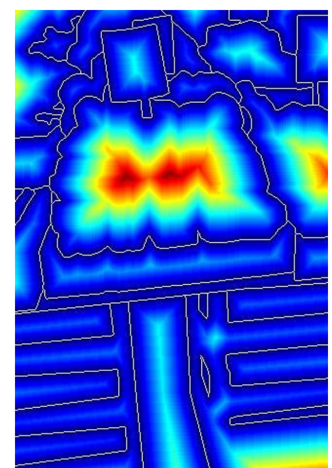
GT5 = GT5_1 + GT5_2



g_1



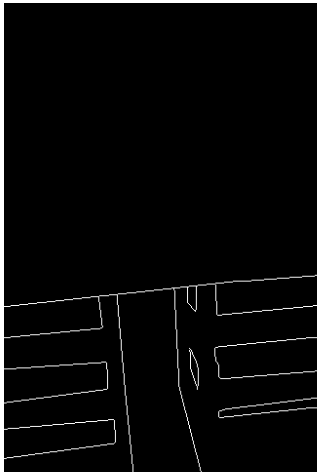
g_2



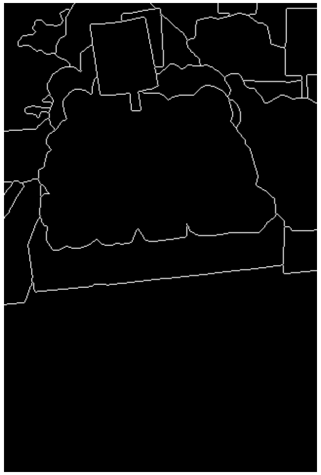
$\inf(g_1, g_2)$

The distance function of the union(sum) is the inf of the distance functions

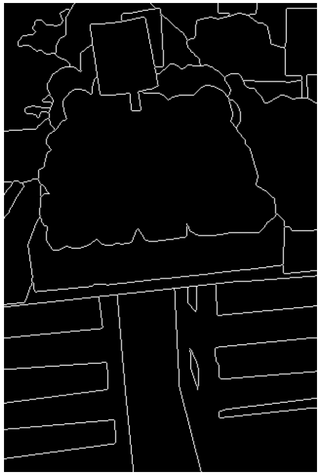
Composition of ground truths:



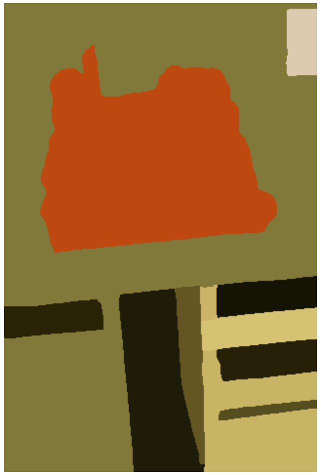
GT5_1



GT5_2



GT5 = GT5_1 + GT5_2



$\omega_{\mathrm{GT5\_1}} + \theta_{\mathrm{GT5\_1}}$

$\omega_{\mathrm{GT5\_2}} + \theta_{\mathrm{GT5\_2}}$

$\inf(\omega_{\mathrm{GT5\_1}} + \theta_{\mathrm{GT5\_1}}, \omega_{\mathrm{GT5\_2}} + \theta_{\mathrm{GT5\_2}})$

Problems

1. Given a hierarchy H and ground truth partition G find the partition in H closest to G.
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2. Compare any hierarchy H with multiple ground truth partitions of the same image
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Global Precision-Recall Energies

The two half distances yield two local and then two global energies:

- Precision (P) : How close is on average the ground truth to the class (G->S)
- Recall (R) : How close is on average the Class contour to the Ground truth (S->G)

$$\widetilde{\omega}_G(S) = \frac{1}{\partial S} \int_{\partial S} g(x) dx$$

$$\widetilde{\theta}_G(S) = \frac{1}{G \cap S} \int_{G \cap S} g(x, \partial S) dx$$

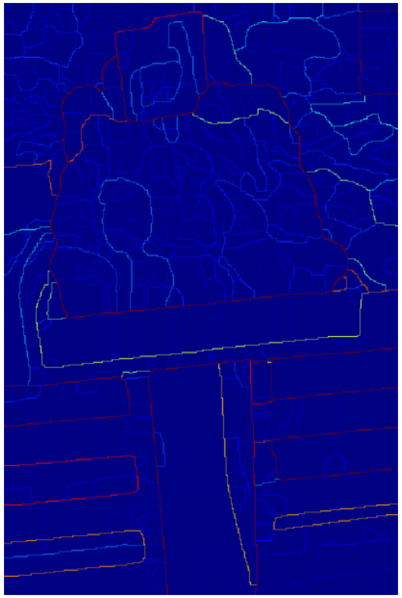
Local dissimilarity measure

$$P = \sum_{i=0}^{1} \frac{i}{N} \frac{\int_{x \in \epsilon(S_i)} (1 - g(x)).S_i(x) \mathrm{d}x}{|S_i|}$$

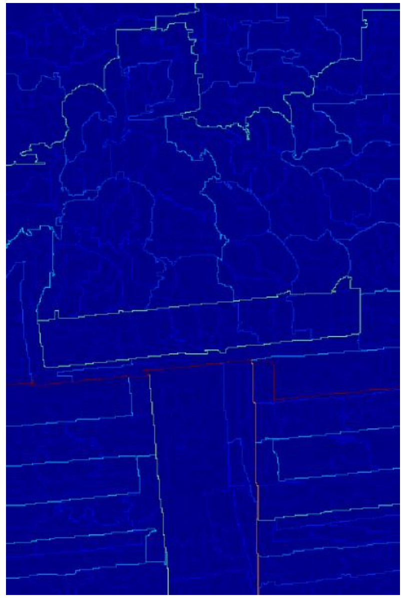
$$R = \sum_{i=0}^{1} \frac{i}{N} \frac{\int_{x \in G} (1 - g_{S_i}(x)) \mathrm{d}x}{|G|}$$

Counterpart Global similarity measures

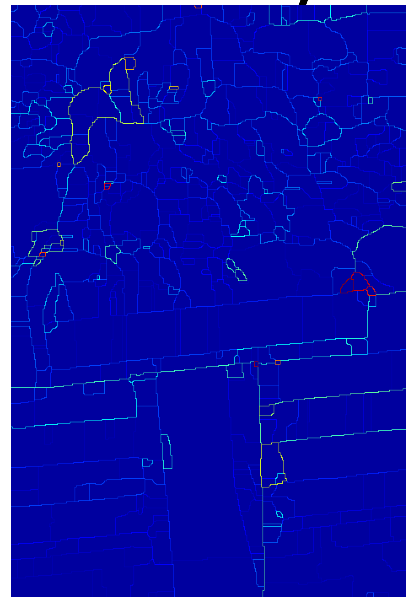
Comparing Hierarchies (saliencies) with Precision-recall similarity measures



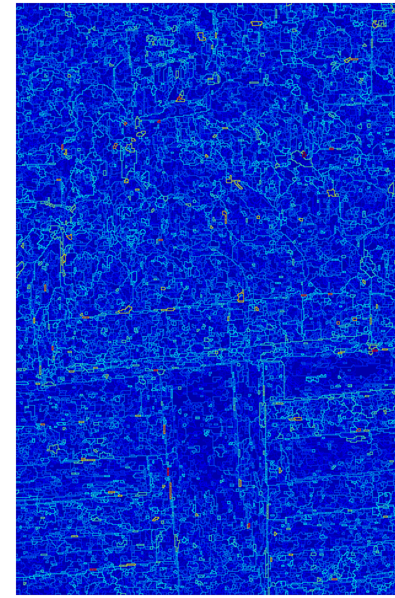
UCM



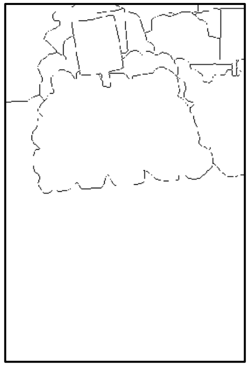
Cousty
(floodings of watershed)



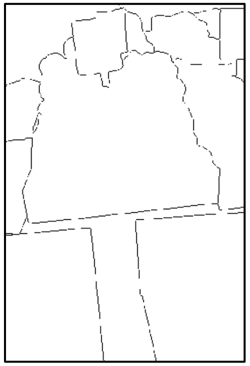
UCM random hierarchy



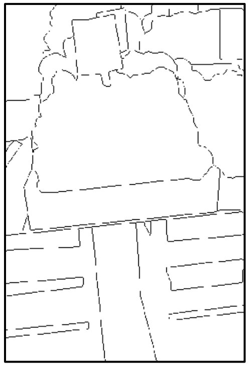
Cousty random hierarchy



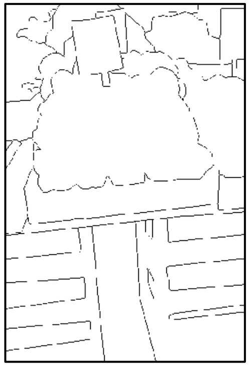
GT1



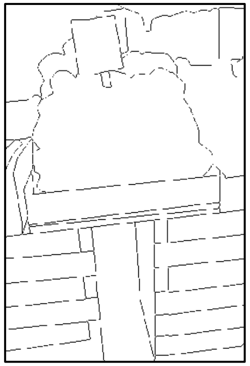
GT2



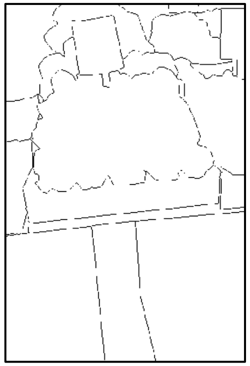
GT3



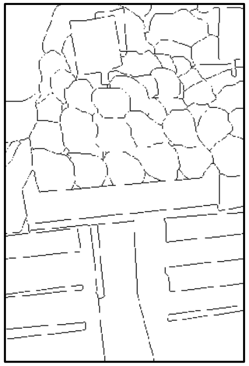
GT4



GT5



GT6



GT7

Comparing Hierarchies (saliencies)
with Precision-recall similarity measures

Image 25098	**UCM**	**UCM random**	**Cousty**	**Cousty random**
Precision energy	4.4	0.27	0.13	0.09
Recall energy	3.9	0.28	0.16	0.10

Integrals from PR equations expressed per 1000 pixels in the image

Problem context

1. Developing the theory of optimal cuts.
   (Pattern Recognition Letters Journal 2013)

2. Ground truth energies (ISMM 2013)

3. Saliency transforms (SSVM 2013)

Ground truth: Evaluation of Hierarchies

Input Image G_2 G_7

Hand drawn ground truth by multiple users or experts for each image.
No inclusion ordering assumed in the ground truths.

A problem: Transforming hierarchies

- Classically the Ground truth is a model to evaluate a given hierarchy of segmentations H.

- But conversely could the ground truth be used to modify and improve the hierarchy itself ?

- If a hierarchy is characterized by its saliency s, how to synthesize a new saliency that incorporates the ground truth?

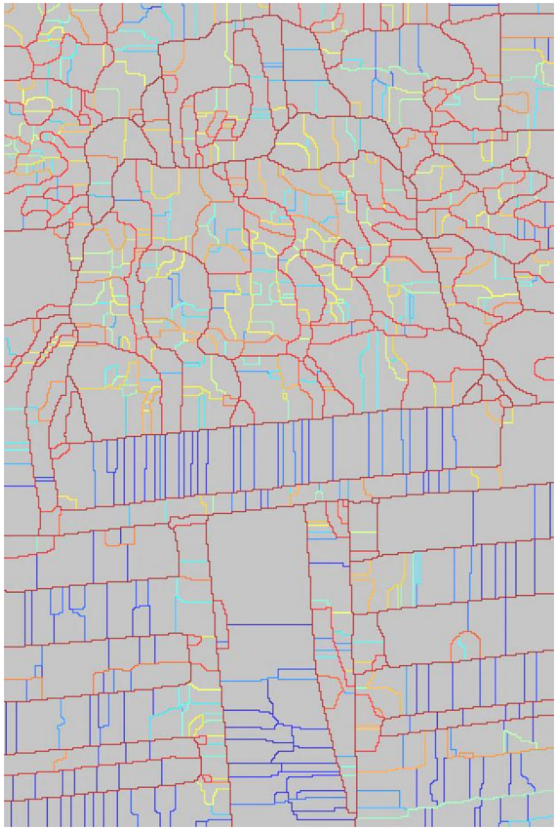
- Can we generate a hierarchy based on the proximity to the ground truth?

Representations of Hierarchies: Saliency function

1. Weighting function associated with the edges between classes of hierarchy H.

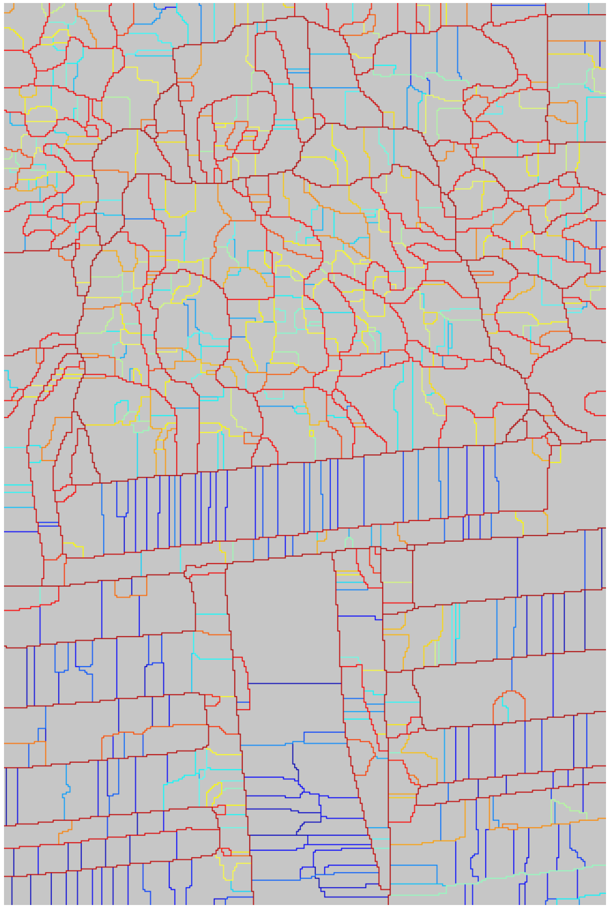
2. For a given edge, this function, constant along the edge, is the level of H when the edge disappears.

3. Clearly, a distribution of arbitrary weights on the edges may not be saliency. It is also required that by removing one edge one still maintains a partition, i.e. that one does not create pending edges.



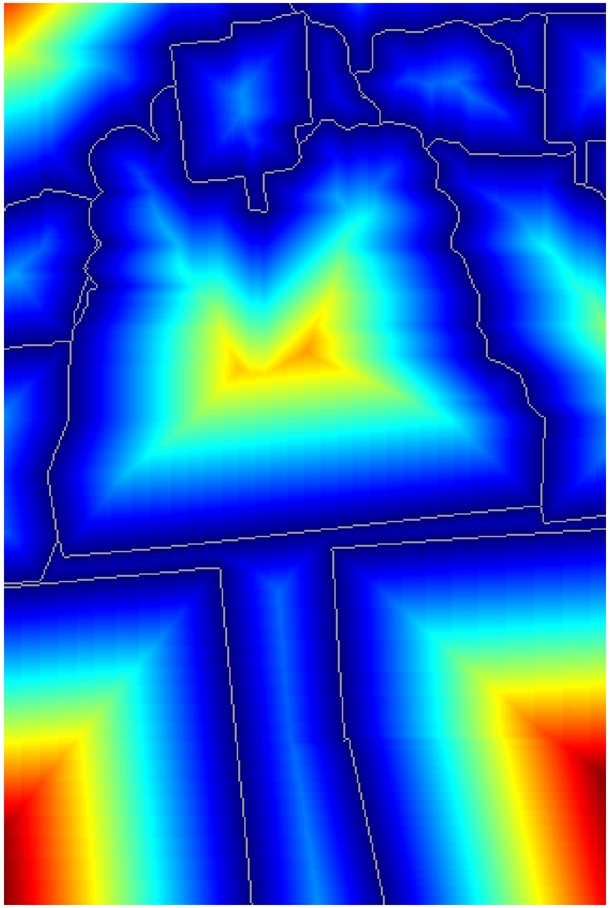
Saliency
Ultrametric contour Map (UCM)

Introducing an external function



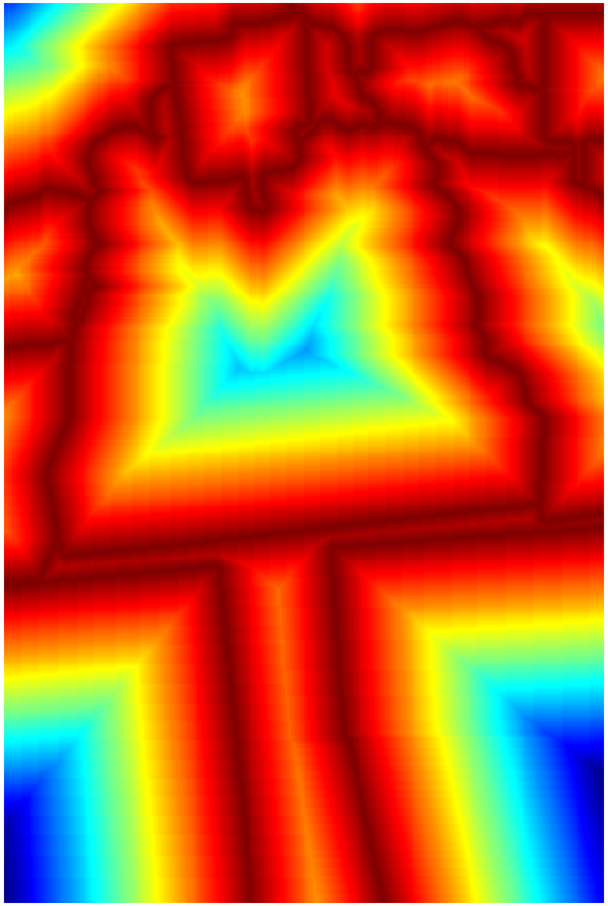
s

Saliency



g

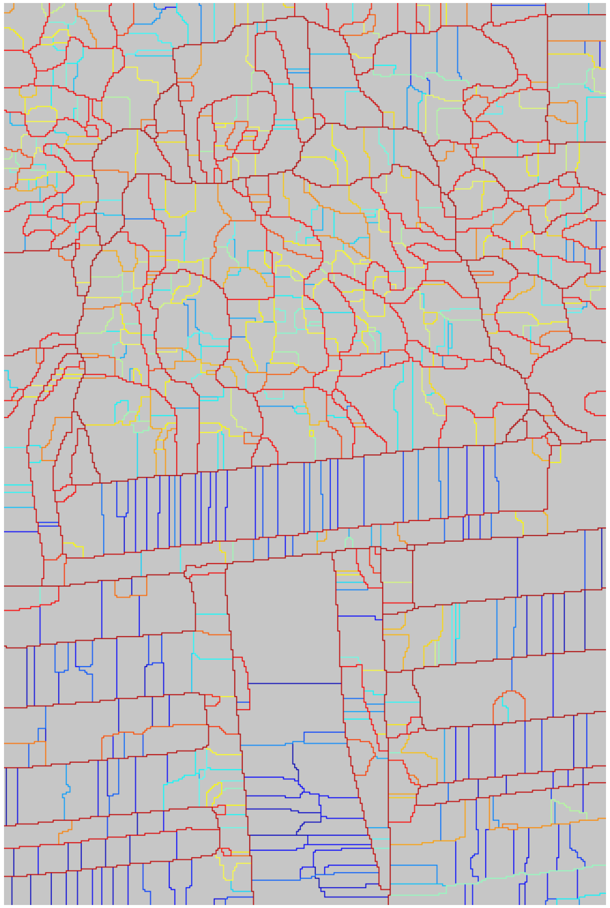
Ground truth distance function



$max(g) - g$

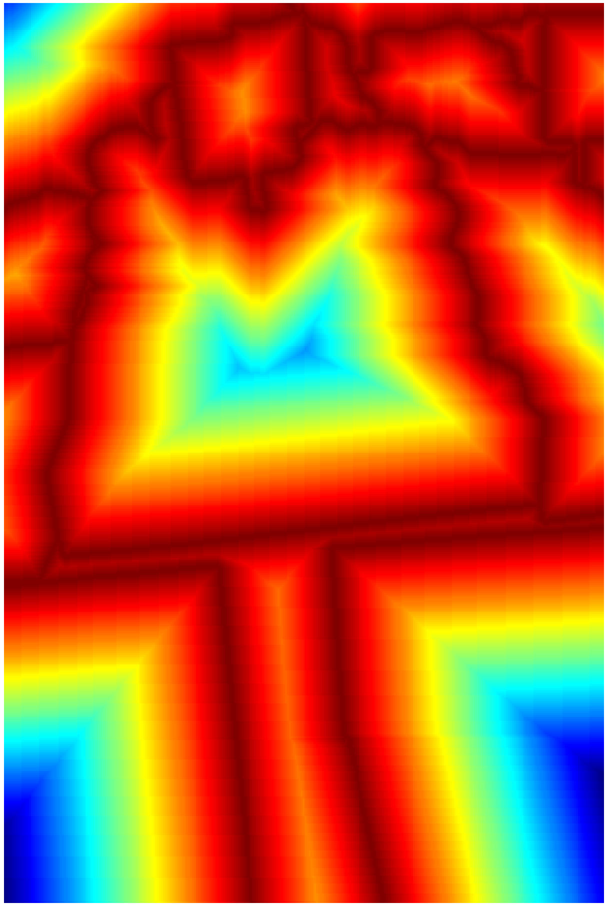
Inverted distance function

Introducing an external function



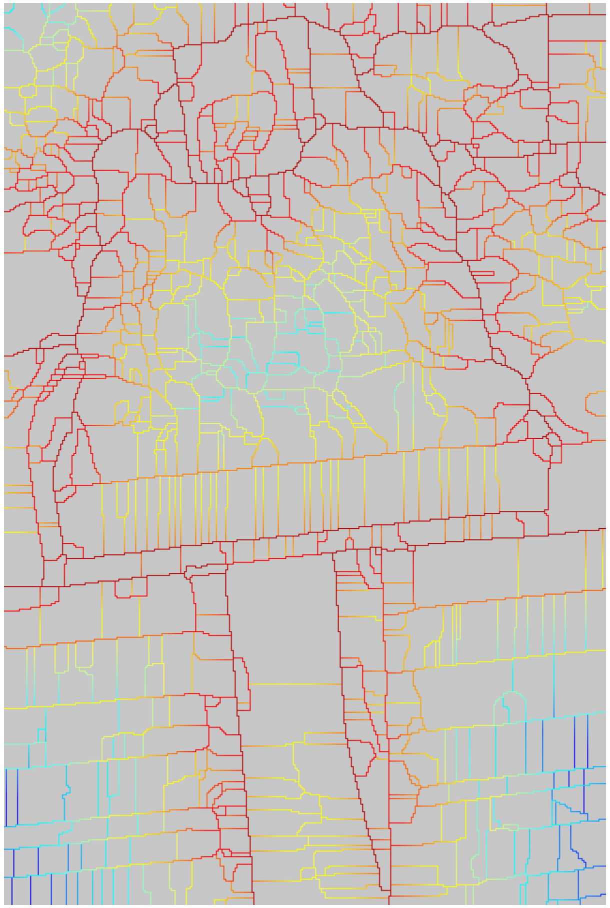
s

Saliency



$max(g) - g$

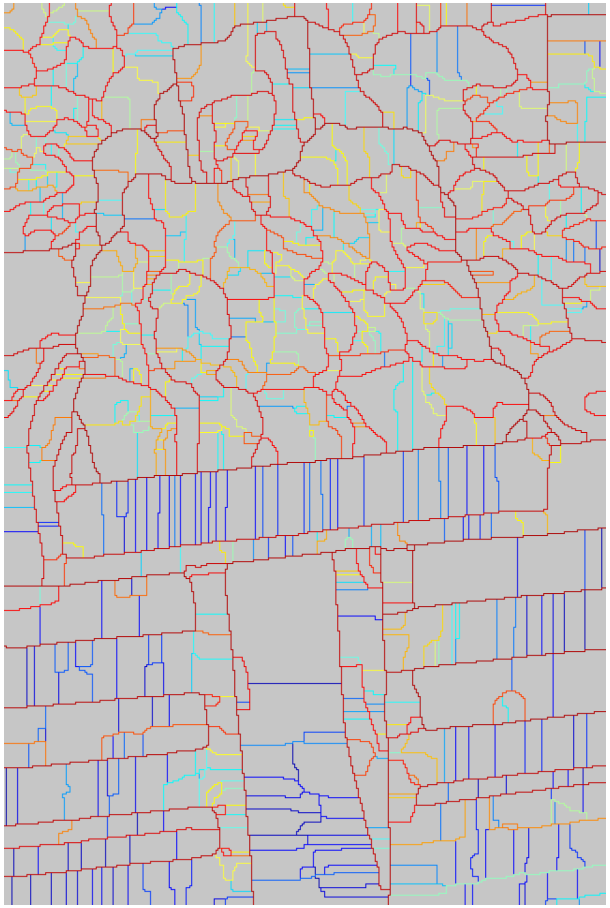
Inverted distance function



$\varphi = s + max(g) - g$

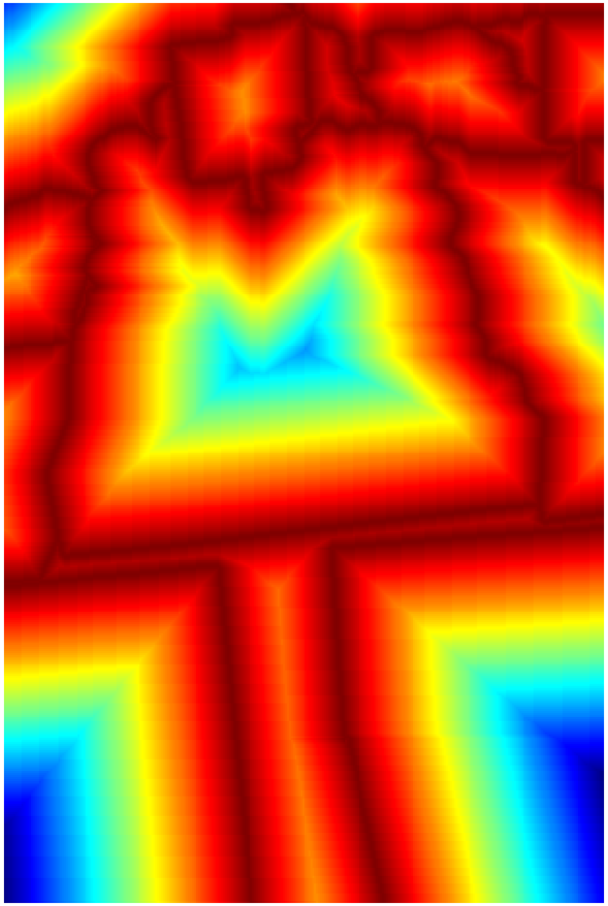
Similarity Function

Introducing an external function



s

Saliency



$max(g) - g$

Inverted distance function

$\varphi = s + max(g) - g$

Similarity Function

Binary Class Opening

Given a finite set of simple arcs $\mathcal{P}(E_0)$ in 2D space E, we define

$$\gamma : \mathcal{P}(E_0) \to \mathcal{P}(E_0)$$

$\gamma(X)$ reduces each set of arcs $X \in \mathcal{P}(E_0)$ to the closed contours it may produce.

Theorem *the operation* $\gamma : \mathcal{P}(E_0) \to \mathcal{P}(E_0)$ *is an opening.*

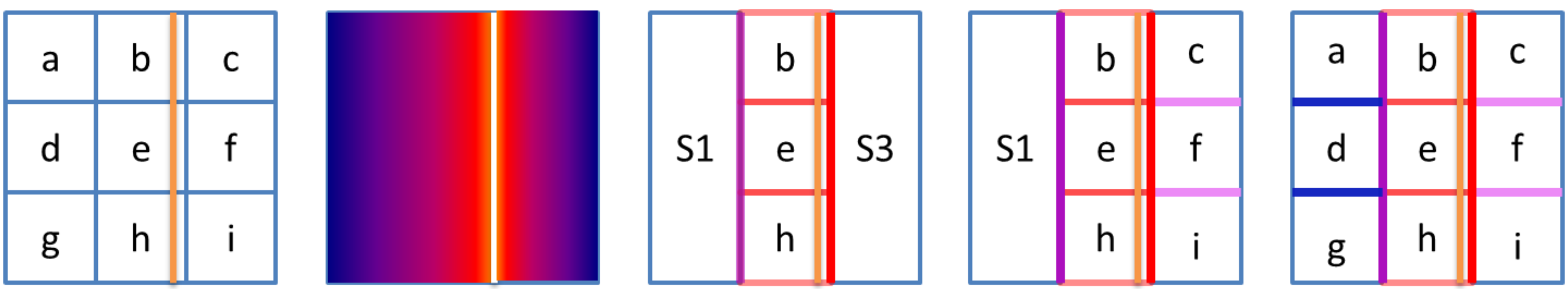
Numerical (grayscale) class opening

The numerical extension of γ the class opening, holds now on a numerical function φ on the edges of the leaves.

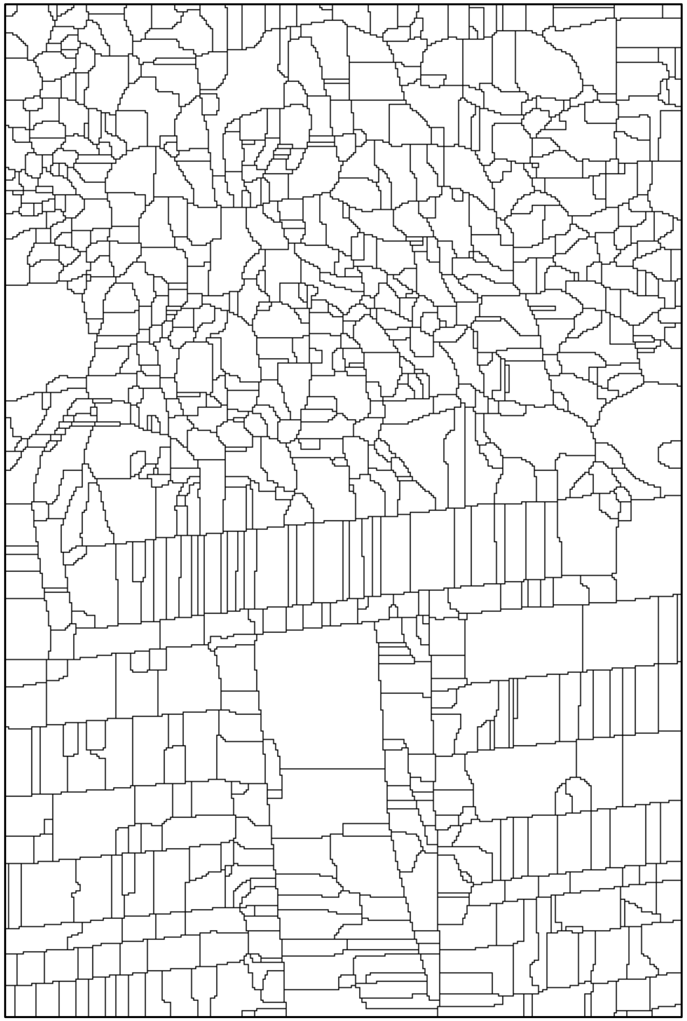
$X_t(\varphi) = \varphi \geq t$, and we define the numerical opening $\gamma(\varphi)$ by its level sets $X_t[\gamma(\varphi)]$ by putting

$$X_t[\gamma(\varphi)] = \gamma[X_t(\varphi)], \quad t > 0.$$

When φ spans the class of all positive functions, then $\gamma(\varphi)$ produces all possible saliencies.

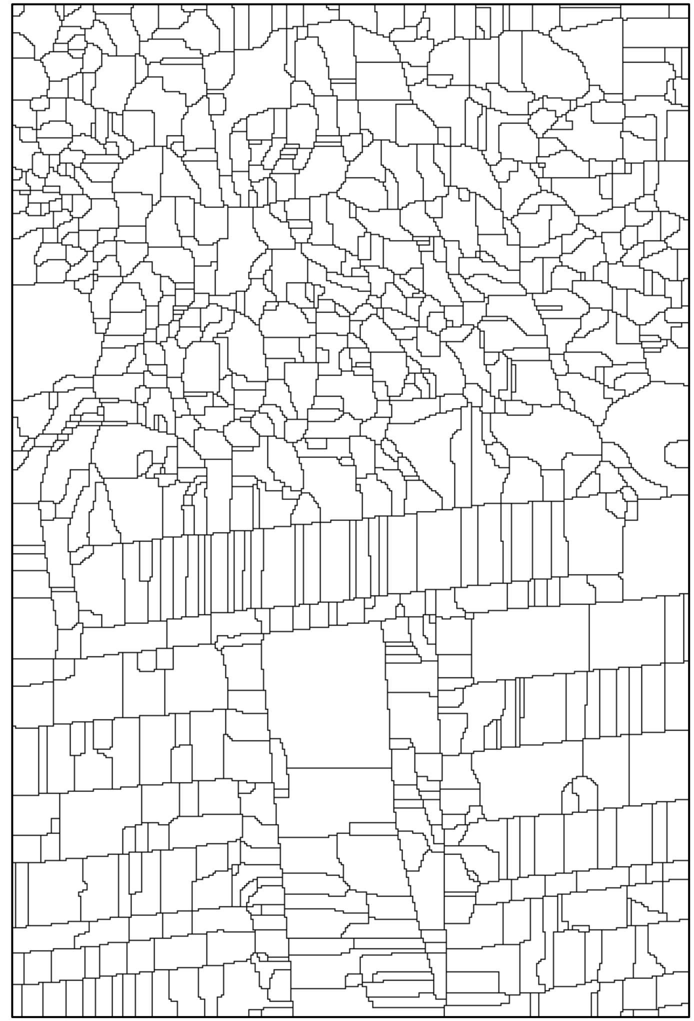


Example: Class Opening



Thresholded similarity function

φ_t



Class opening

$\gamma(\varphi_t)$

Properties of class opening I

Let g_1 and g_2 be two positive functions on $\mathbb{R}^2$ or $\mathbb{Z}^2$, then:

i) $\gamma(g_1)$ (resp. $\gamma(g_2)$) is the largest saliency under g_1 (resp. g_2);

ii) $\gamma(g_1) \vee \gamma(g_2)$ is the largest saliency whose value at each edge is under that of $\gamma(g_1)$ or $\gamma(g_2)$;

iii) if $g_1 \circledast g_2$ denotes an operation from $\mathcal{G} \times \mathcal{G} \rightarrow \mathcal{G}$, such as $+, -, \times, \div, \vee$,or $\wedge$, then $\gamma(g_1 \circledast g_2)$ is the largest saliency under $g_1 \circledast g_2$ and in particular,

$$\gamma(g_1 \vee g_2) \leq \gamma(g_1 + g_2)$$

In all cases the resulting saliency is unique.

Properties of class opening II

Given an input saliency function s, and 3 external positive functions g_1, g_2, g_3

$$s = \gamma(s) \leq \gamma(s + g_1) \leq \gamma(s + g_1 + g_2) \leq \gamma(s + g_1 + g_2 + g_3)$$

The same can be applied for the difference operations if the similarity function representing this difference remains positive (doesn't introduce zeros).

And similarly for the supremum:

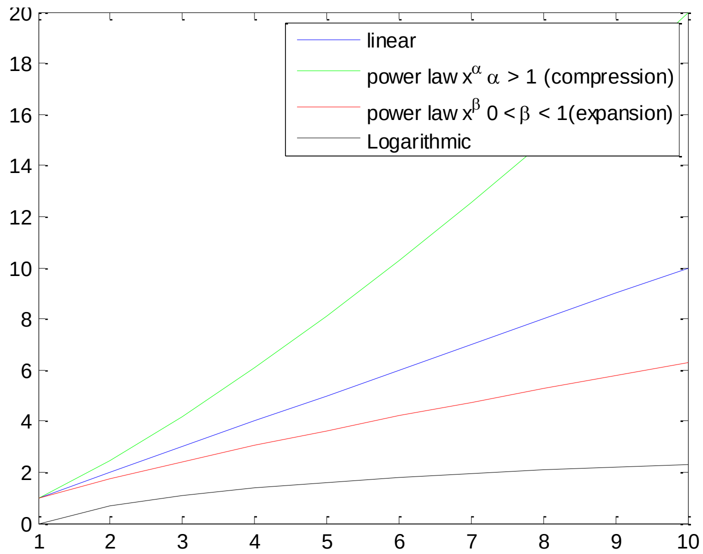
$$s = \gamma(s) \leq \gamma(s \vee g_1) \leq \gamma(s \vee g_1 \vee g_2) \leq \gamma(s \vee g_1 \vee g_2 \vee g_3)$$

All these class openings are ordered thus they form granulometric semigroups.

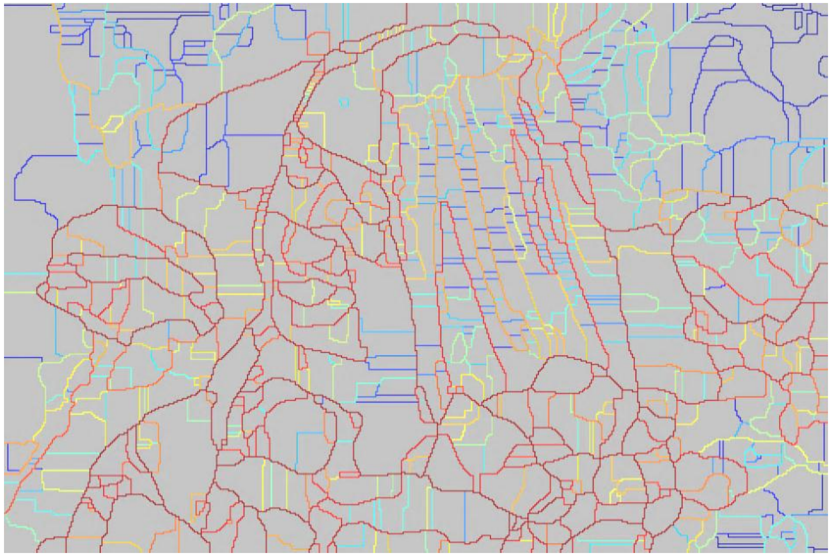
Saliency degeneracy

Class opening $\gamma(\varphi)$ orders φ to obtain a saliency, which corresponds to a hierarchy H_φ.

Degeneracy: Any strictly increasing mapping of the grey levels $\varphi' = \alpha(\varphi)$, e.g. square root, log, etc., yields a $\gamma(\varphi')$ that generates the same hierarchy $H_{\varphi'}$ as $\gamma(\varphi)$ does.

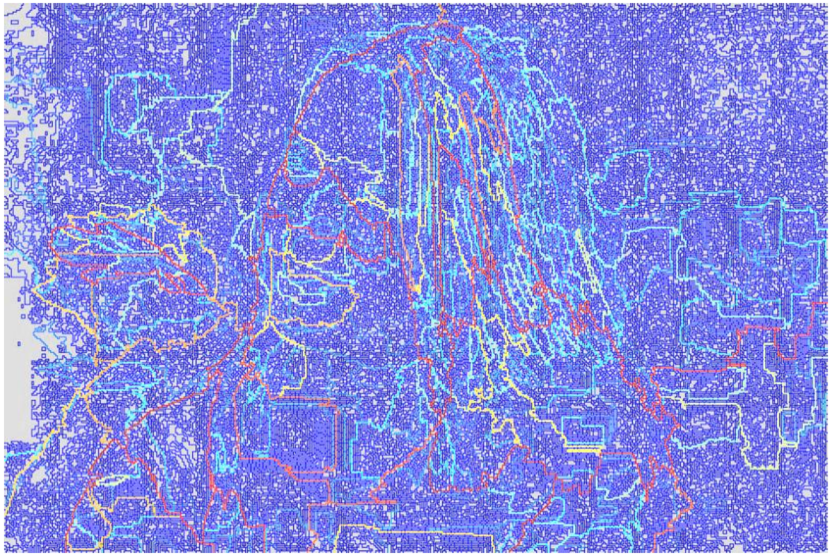


Evaluating Hierarchies w.r.t Ground Truth



s_1

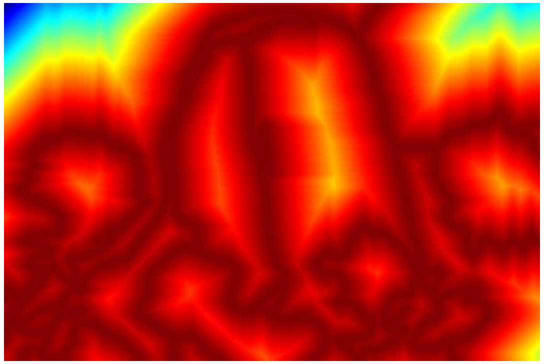
Ultrametric Contour Map



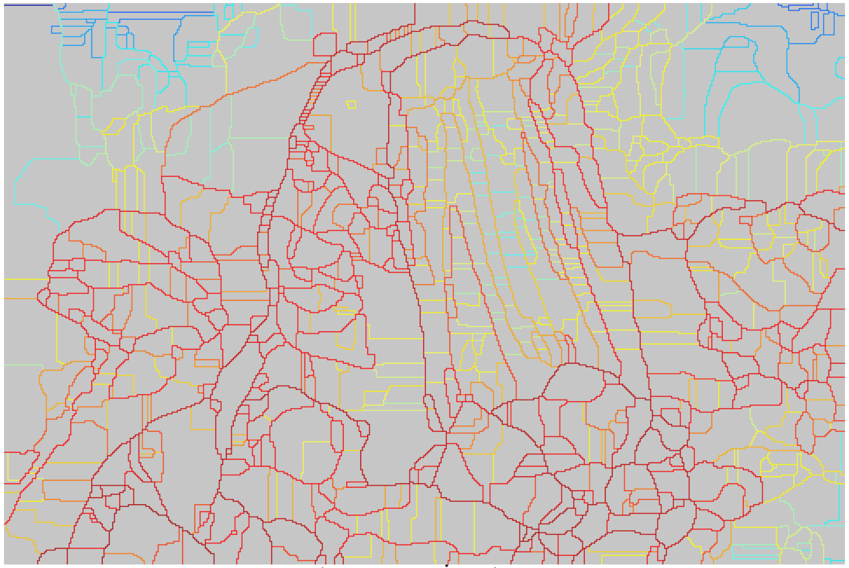
s_2

Volume attribute based watershed flooding

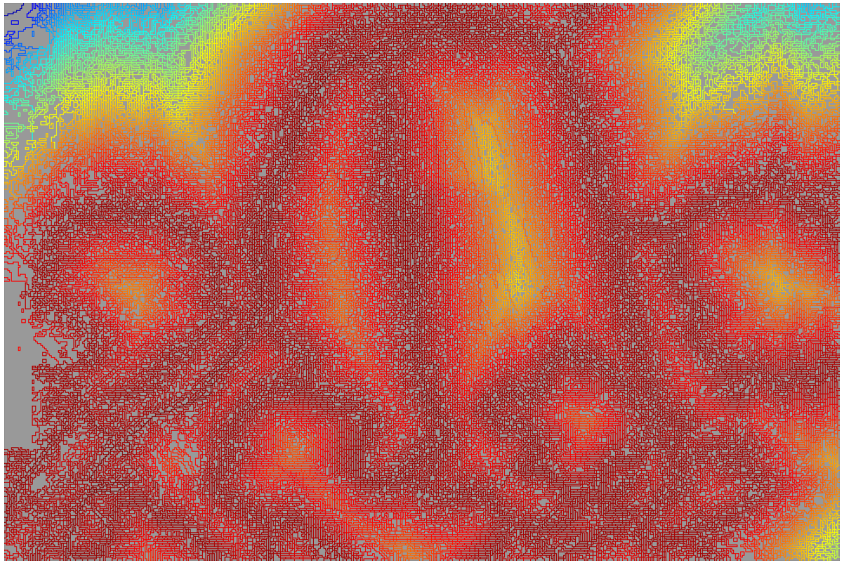
Evaluating Hierarchies w.r.t Ground Truth



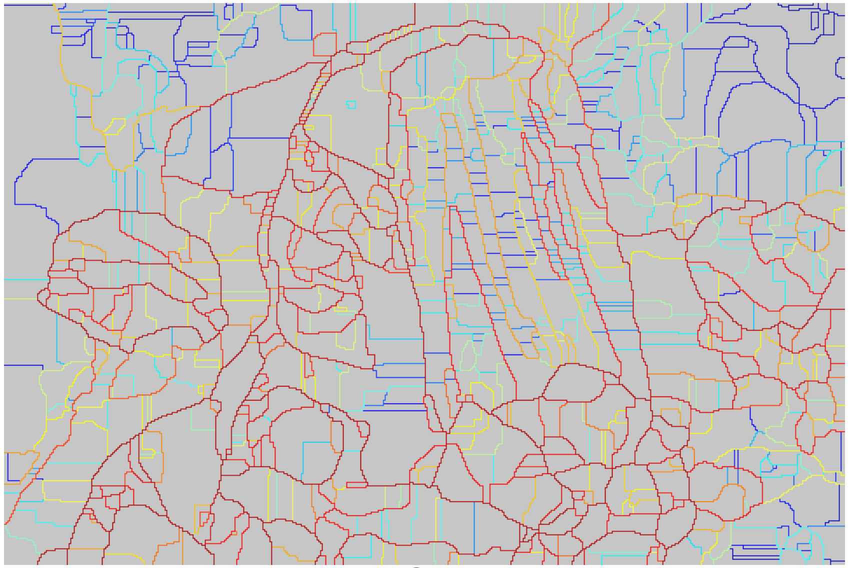
g^{inv}



$\gamma(s_1 + g^{inv})$

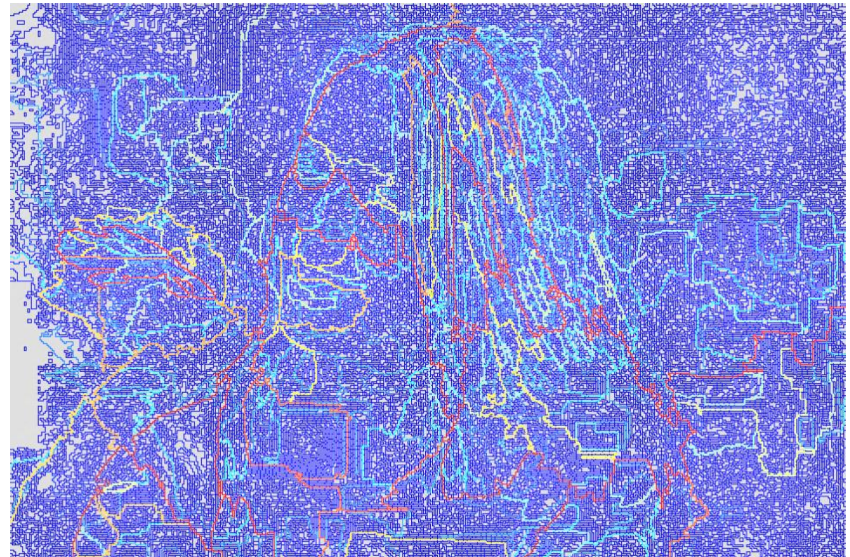


$\gamma(s_2 + g^{inv})$



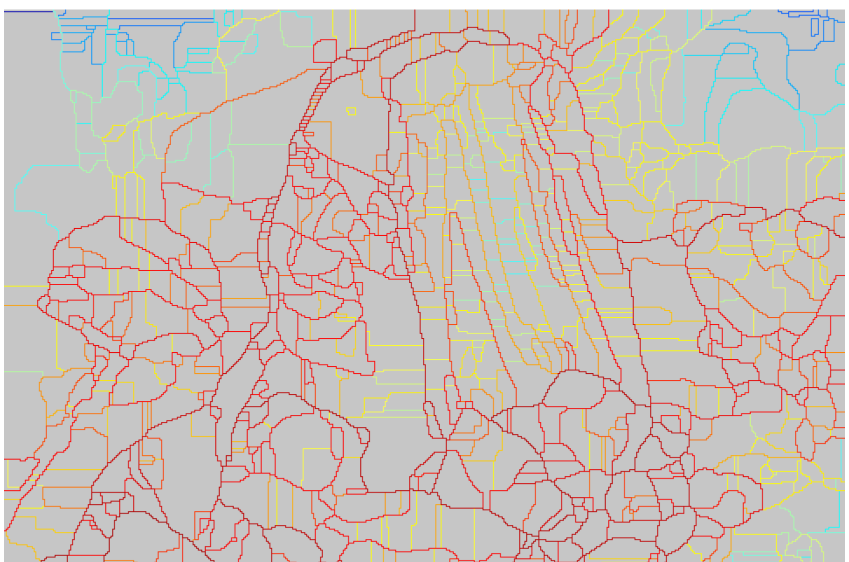
s_1

Ultrametric Contour Map

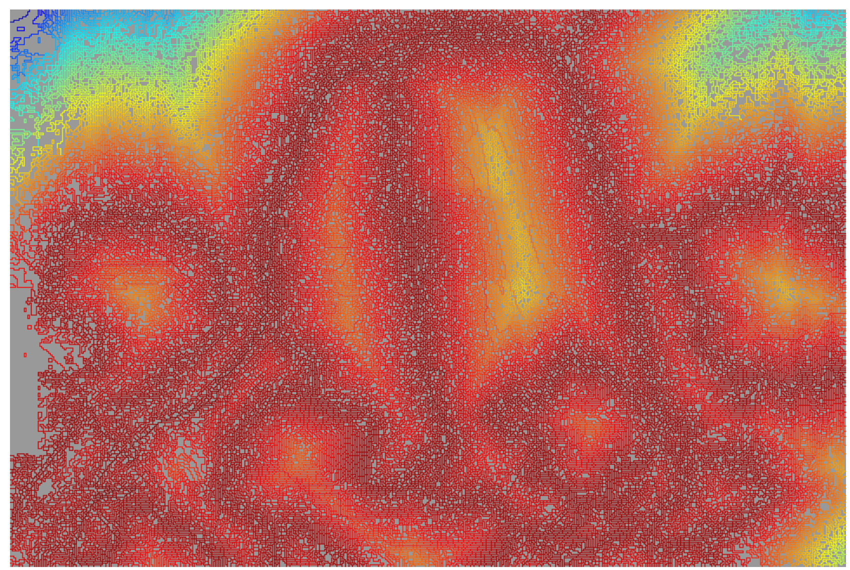


s_2

Volume attribute based watershed flooding

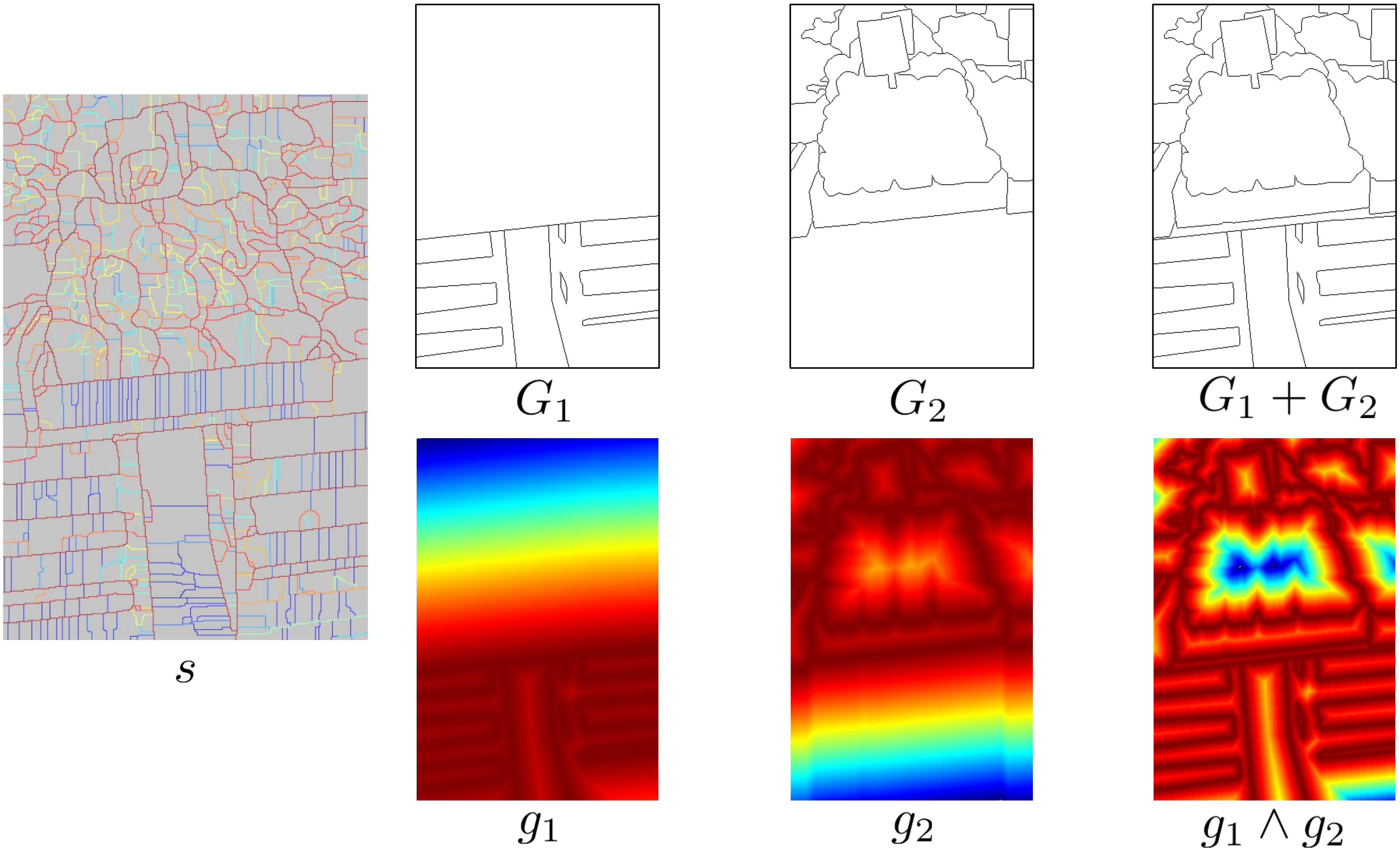


$\gamma(s_1 + g^{inv})$

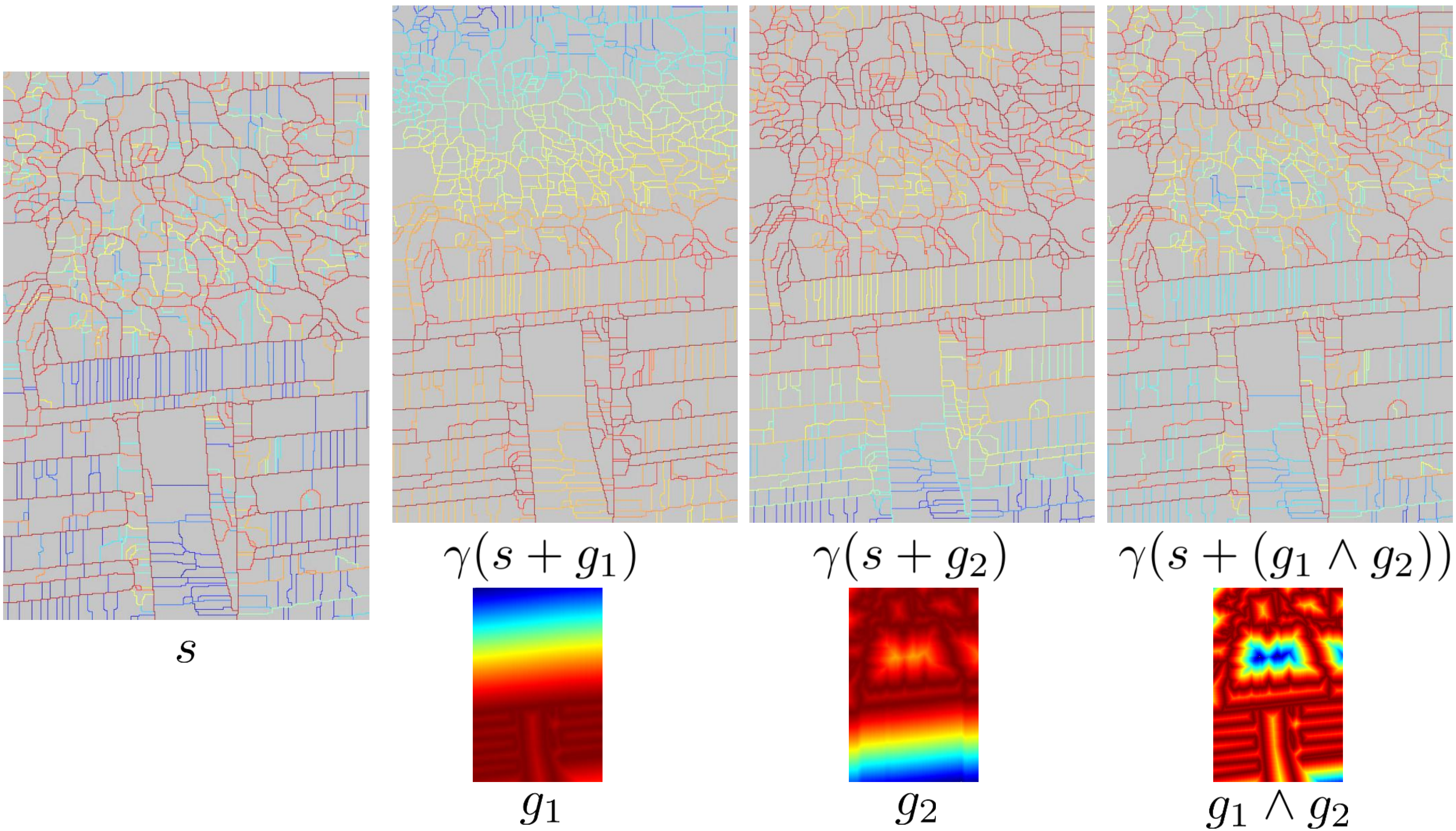


$\gamma(s_2 + g^{inv})$

Composing two external functions

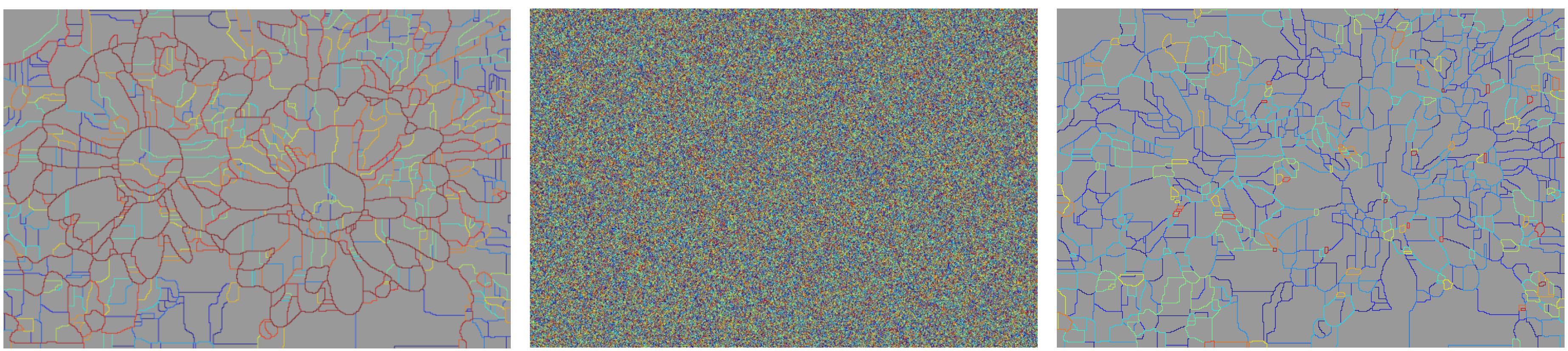


Composing two external functions

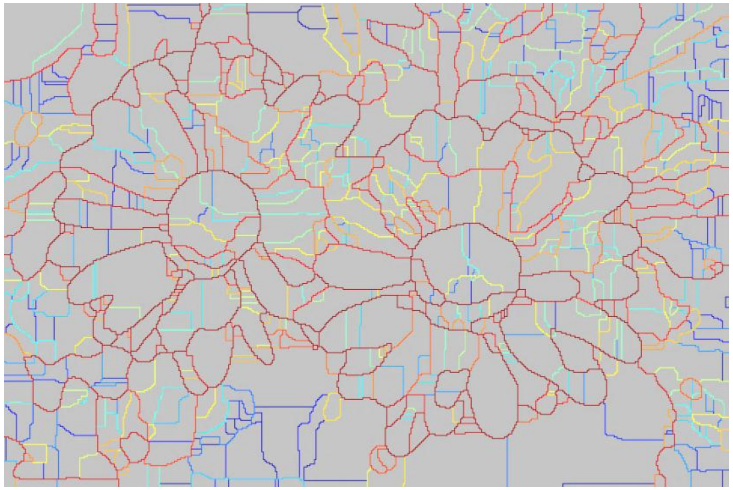


Generating Random Hierarchies

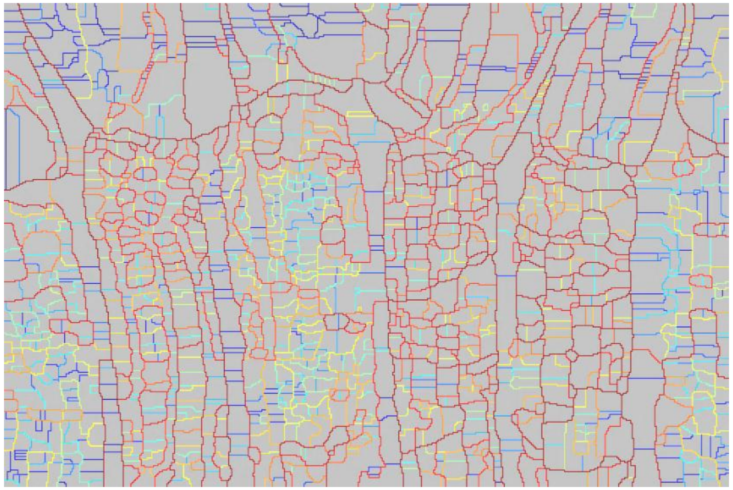
Creating random hierarchies using random permutation matrices as external function



Hierarchy Fusion: Matching hierarchies

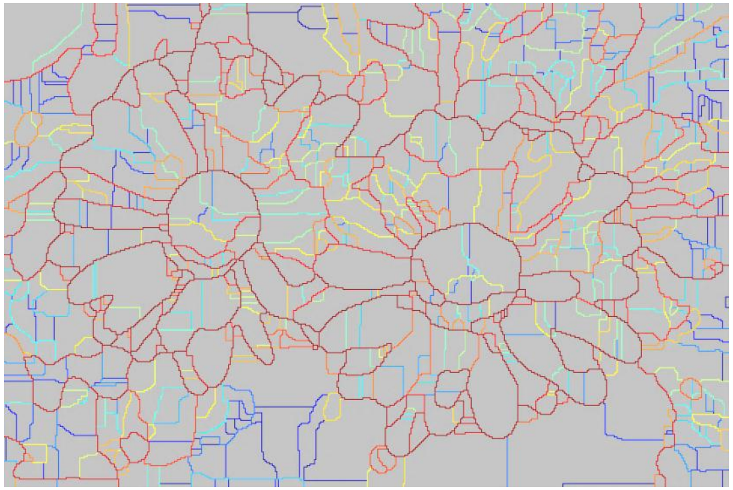


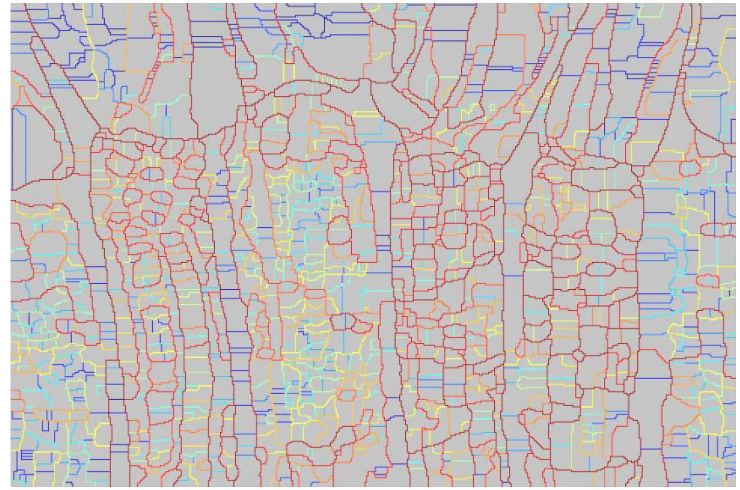
s_1



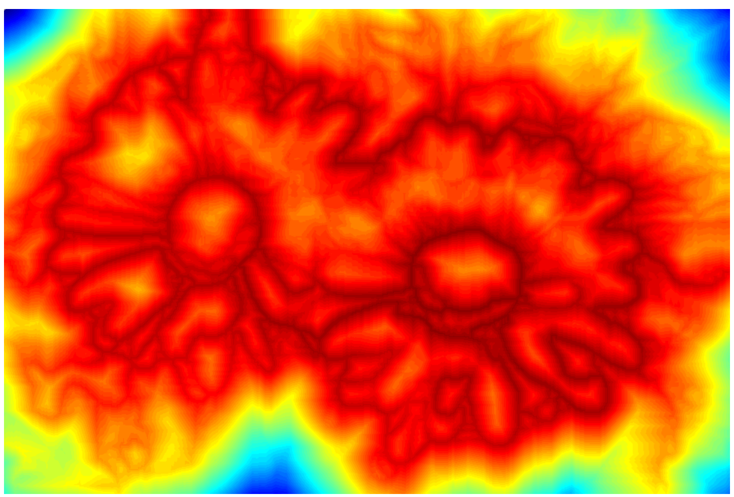
s_2

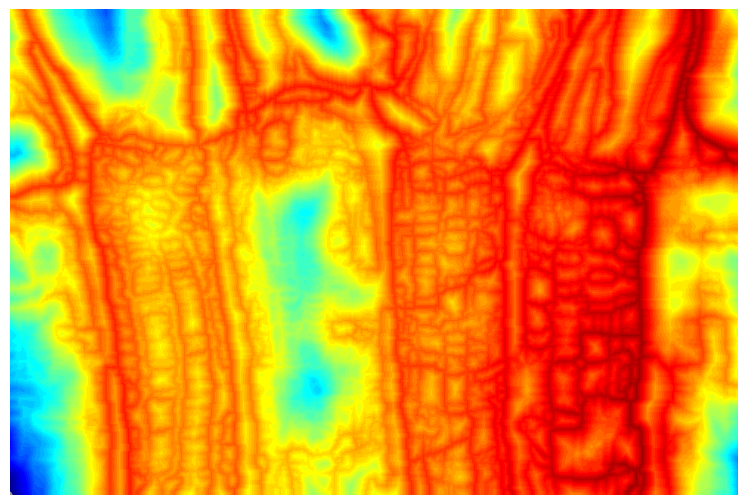
Hierarchy Fusion: Matching hierarchies





s_1 $\quad$ s_2

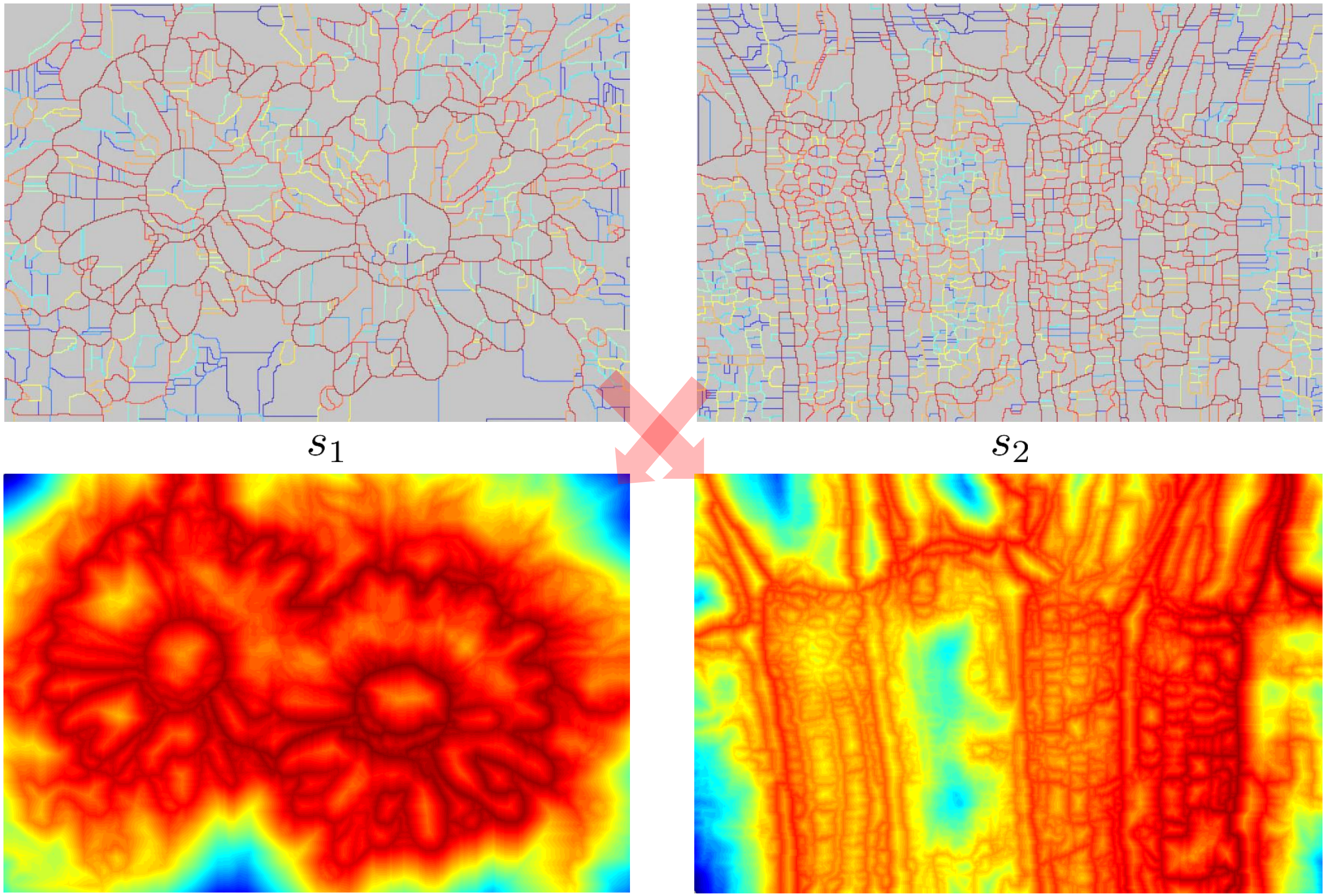




$$d_{\sum}(s_1) = \sum_{t \in range(s_1)} d(s_1 \geq t)$$

$$d_{\sum}(s_2) = \sum_{t \in range(s_2)} d(s_1 \geq t)$$

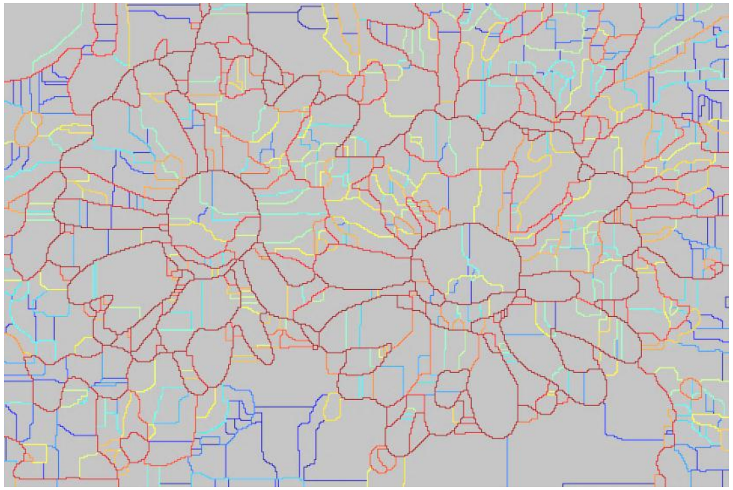
Hierarchy Fusion: Matching hierarchies



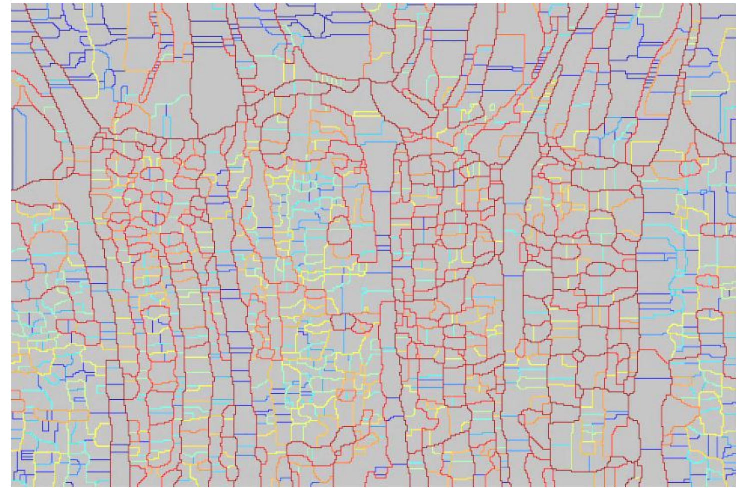
$$d_{\sum}(s_1) = \sum_{t \in range(s_1)} d(s_1 \geq t)$$

$$d_{\sum}(s_2) = \sum_{t \in range(s_2)} d(s_1 \geq t)$$

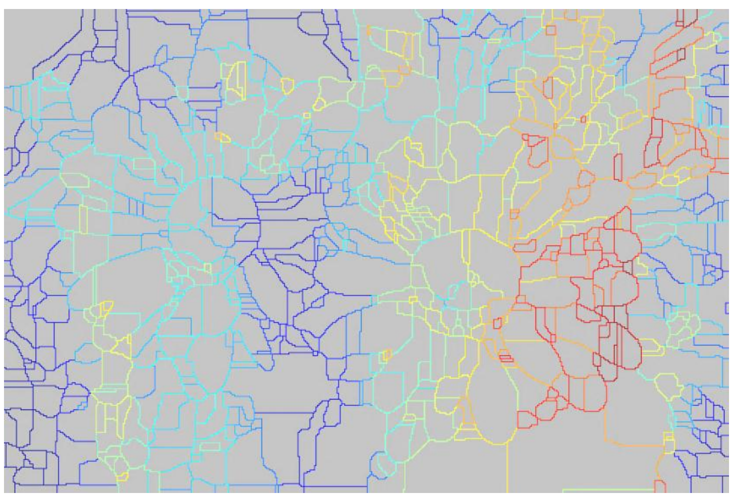
Hierarchy Fusion: Matching hierarchies



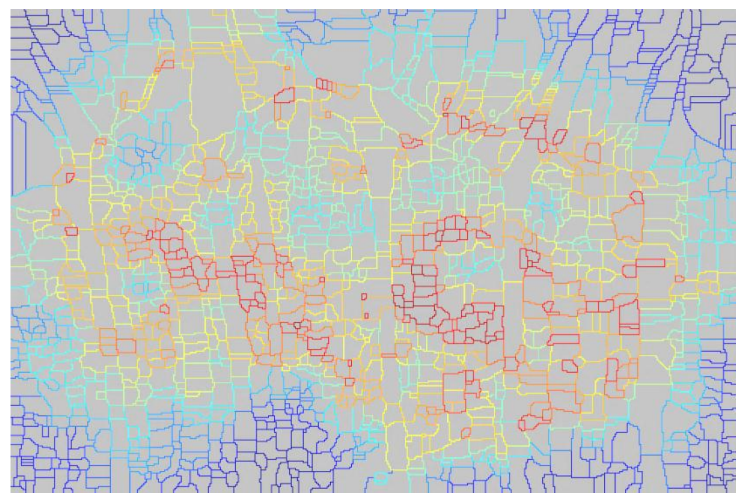
s_1



s_2



$s_{12} = \gamma(d_{\sum}(s_2) + s_1)$



$s_{21} = \gamma(d_{\sum}(s_1) + s_2)$

Conclusion

- Generation of family of saliencies using the *Class opening* $\gamma(s)$ by composition with external function g.

  Results for ground truth distance function.

- Composition of multiple external functions.

- Fuse two or more hierarchies (saliencies).

Code will be available shortly here: http://www.esiee.fr/~kiranr/HierarchEvalGT.html

Future work

- Develop the converse approach where we interchange the roles of saliency and the ground truth.
- Define energies which yield significant optimal cuts.
- Analyse the changes in dendrograms under saliency transformation.
- Introduce conditional saliency transform based on attributes like volume, area, dynamic.
- Use the approach for time varying hierarchies.

Merci beaucoup pour

- Votre patience
- Et votre attention

Avez vous des questions ?